Designing for Embodied Sense-making of Mathematics: Perspectives on Directed and Spontaneous Bodily Actions

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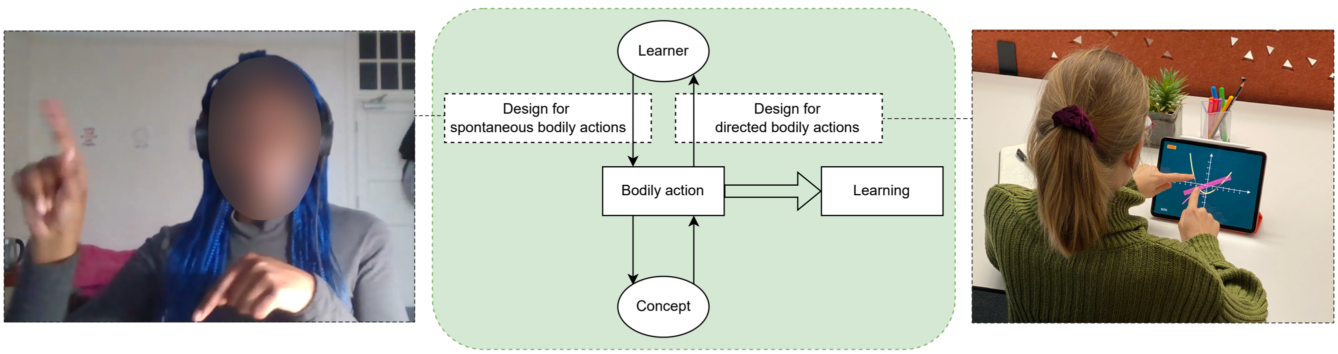

Figure 1: When making sense of mathematical content, students perform bodily actions: either spontaneous bodily actions, performed spontaneously while solving a task (right), or directed bodily actions, performed as requested by a task (left).

ABSTRACT

While mathematics is conventionally viewed as an abstract discipline, contemporary perspectives on embodied cognition underscore the significance of integrating students' bodily experiences into the learning process. However, the efficacy of embodied learning activities, as compared to traditional methods, remains under scrutiny. We argue that both directed and spontaneous bodily actions should be considered when designing embodied learning activities, and explore such bodily actions through two studies. A quantitative user study involving directed bodily actions in Virtual Reality and on tablet reveals VR's support for math-anxious and body-aware learners, and distinct movement patterns related to varying mathematical abilities. A subsequent qualitative analysis identifies key characteristics of spontaneous bodily actions, namely coarseness, muscle tension, repetitions, anchors, perspective, and metaphors. Derived from both studies, we propose design recommendations, advocating for expanded embodied interaction design, consideration of embodied metaphors, coarse gesturing for deep features identification, supporting of sense-making anchors, and in-VR learning assessments.

CCS CONCEPTS

• **Human-centered computing → Human computer interaction (HCI)**; **Empirical studies in HCI**; **Virtual reality**; *User studies*; *Gestural input*.

KEYWORDS

embodied interaction; embodied cognition; embodiment; mathematics education; virtual reality; problem solving followed by instruction

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1 INTRODUCTION

Although it is often considered as the mere shadow of imperceptible ideals, researchers argue that mathematics rather comes from our embodied relationship with the world. Our sensory experiences inform mathematical theories and concepts. For example, the notion of infinity finds parallels in our everyday attempts to comprehend things larger than our immediate perception, such as the vastness of the universe or a never-ending process [62]. In this light, embodied cognition is a set of theories presenting cognition as not confined to the mind alone; it unfolds within the context of our physical bodies and the environments we inhabit [3, 103]. This approach considers thinking a form of truncated action: a physical action is planned and simulated but not externally expressed through muscle engagement [70].

Given that many traditional activities for learning mathematics are disembodied, recent research explores how to meaningfully integrate movement and bodily actions into learning activities [99]. Meanwhile, the widespread of novel technologies, particularly those emphasizing bodily engagement such as Virtual Reality (VR), offers exciting perspectives supporting this transition [71, 72]. As such, when designing interaction with these technologies, we ought to ask what specific bodily actions support learning and how learners make sense of the underlying mathematical concepts through them.

In this work, we explore the design space of embodied interaction in the context of embodied learning activities for mathematics [18], focusing on bodily actions specifically. To do so, we adopt a learner-centered approach and provide an in-depth exploration of bodily actions performed when making sense of mathematics in embodied learning activities. We focus on two types of bodily actions: *directed* bodily actions — those explicitly instructed per the embodied learning activity — and *spontaneous* bodily actions — those spontaneously generated by learners as they make sense of the mathematical concept. We argue that both should be considered in design and, as such, we present two respective user studies to identify the key characteristics of these bodily actions and address our research questions:

RQ1 How do students move when making sense of derivatives in an embodied activity focused on directed bodily actions?

RQ2 Which bodily actions do individuals spontaneously perform when making sense of derivatives?

We then discuss design implications accounting for the diversity of learners and detail potential avenues for future research.

2 RELATED WORK

In this section, we provide a succinct overview of embodiment literature and position our approach. We then provide background information for the individual factors explored in our project.

2.1 Embodiment

The embodied cognition view stipulates that cognition is not simply located in learners' minds and rather extends to their entire bodies [3, 94]. For the sake of this paper, we use the following working definition for embodied cognition: the view that our cognition is shaped by the features of our bodies and environments [66]. As cognition is deeply involved in learning and interaction, it follows that our bodies play a crucial role in both, which we detail below.

2.1.1 *Embodied learning.*

While there are different forms of the embodied cognition theory and not all scholars agree on its implications, there is overall undeniable proof that our bodies play a role in learning [39, 69, 103]. Generally, embodied learning is concerned with the design of learning activities acknowledging the embodied cognition view, and therefore integrating learners' bodies and environments in the learning process. Embodied learning has been explored in various domains [6, 99], such as mathematics [21, 45, 78, 97], but also computer science and data science [7, 14], AI literacy [32], chemistry [75], cultural heritage awareness [24], dance [56], and much more.

The process of embodied learning relies on three main mechanisms of embodied cognition [59]. First, stimuli trigger *sensorimotor simulation* and reenactment of past experiences, which is facilitated by gestural congruency. Second, certain bodily actions impact learners' states and feelings, independently of other cognitive mechanisms, through *direct state induction*. Third, *modal priming* impacts the accessibility of concepts once associated to certain bodily states.

Specifically, this means that the design of embodied learning activities often relies on multi-modality for cognitive offloading, to prevent saturation of learners' working memory [103]. Moreover, embodied learning experiences rely on grounding, i.e. forming a mapping between a concept and a concrete referent [76]. For example, in previous work, we demonstrated grounding graph theory in embodied concreteness, "a form of concreteness that involves a high degree of embodiment, in a situated and relatable context", by representing graphs as pipe systems [21].

In our work, we put the focus on bodily actions supporting embodied learning. As a simple example, let us consider the following situation: Lisa wants to count the number of chocolates in her basket. To do so, she uses her fingers to represent the quantity of chocolates, that is, she uses bodily actions to represent an underlying and more abstract concept. Looking at the chocolates, she extends one finger per chocolate and observes that she has four chocolates (4). Now her parent comes in and puts one extra chocolate in the basket. To consider this new chocolate, she extends another finger (+1) and observes that she now has five extended fingers on her hand (5). By this observation, she may make sense of the underlying concept, which is $4 + 1 = 5$. More generally, a learner performs bodily actions to represent a concept and, in turn, uses this representation to make sense of the concept. This process can be represented as a system [18], reproduced in Figure 2 (left).

During the process of embodied learning, two main types of bodily actions can be involved in embodied learning: *spontaneous bodily actions* and *directed bodily actions*. While this distinction is crucial and has been modeled in great detail in previous work [101], existing design frameworks for embodied learning activities often neglect the synergy of both types of bodily actions. They rather consider the distinction between position-focused and movement-focused or body-focused, object-focused, and environment-focused approaches [80], or explore examples of spatial and temporal embodiment [99]. In our work, we bridge this gap, substantiated by our integration of the two design spaces into the system representation of embodied cognition (Figure 2, right). In the following, we provide representative examples of learning activities involving either type of bodily actions.

First, *spontaneous bodily actions*, or explanatory gestures, are the bodily actions that learners perform spontaneously when making sense or communicating about mathematical content. These gestures are used as artifacts to represent and communicate various mathematical concepts [9, 31], and can contribute to sensorimotor regulation when integrated in learning activities [97]. Importantly, spontaneous gestures are central to meaning-making, and constitute preliminary evidence of learning [86].

Second, *directed bodily actions*, are the bodily actions that learners perform as requested per an activity. For example, when learning choreography, dancers benefit from "marking", a process of deliberately representing choreography steps through smaller representative gestures [57]. Directed bodily actions are also integrated in digital learning activities. In the game "The Hidden Village", learners are explicitly taught gestures to represent concepts of geometry, such as elbow opening to represent angles [78, 96]. In the "Mathematical Imagery Trainer" activity, students are invited to explore proportion problems by moving their hands up and down in order to "keep the screen green", which is achieved by keeping the same ratio between their hands and a reference point [45].

2.1.2 Embodied interaction for embodied learning. While our bodies do influence learning, moving does not necessarily result in learning [99]. Nevertheless, the field of embodied learning suffers from a lack of empirical studies measuring learning outcomes [6]. The rare available quantitative studies fail to show significant learning benefits of embodied learning activities over their counterparts, and often highlight design limitations [6, 19, 21, 67, 73].

We hypothesize that these results are due to a lack of interaction-focused design guidelines for embodied learning. Indeed, when looking into the design of embodied learning activities, especially in digital contexts but not only, considering the interaction with the learning artifacts is also crucial. Generally, embodied interaction acknowledges the embodied view on cognition and stipulates that "interaction is grounded in and informed by its physical and social context" [18, 29]. While previous work identified guidelines for embodied design, these either focus on the overall design of embodied learning activities [3, 4, 49], or reflections on the design of interaction, without considering learning contexts [8, 42, 46, 74].

Nevertheless, these strong concepts and approaches to interaction design should be considered in embodied learning activities. Mueller et al. presented the distinction between the flesh body, *Körper*, and the feeling body, *Leib* [74]. Designing for the *Körper* reduces users as physical entities utilized to press buttons and perform actions. Designing for the *Leib* acknowledges users as feeling beings and benefits from direct state induction. From a learning standpoint, such considerations are important as certain bodily actions may be used to reduce anxiety [47] or offer a sense of embodied achievement by using a "high five" gesture [19] or a "winning pose" [21] to switch to the next level in a math learning game.

In turn, Höök et al. present Somaesthetic Appreciation as a strong concept for the design of embodied interaction through inquiry [42, 43]. With this approach, the resulting designs offer *subtleness* in how they encourage bodily inquiry, require *intimate correspondence* of feedback and interaction synchronized with users bodies, and *make space* for learning and somatic awareness. These lessons are useful for learning as well, as they shift awareness towards one's body and bodily sensations, which could support awareness of embodied learning processes and embodied depictions of concepts.

Klemmer et al. discussed the design of embodied interaction, acknowledging research in embodied cognition and embodied learning, and identified five themes of importance [58]. Considering individual corporeality, we *think by doing* and integrate mind and body to learn, and as physical actions are rich, fast, and nuanced, they support *performance*. Considering social affordances, bodily actions support collaboration through *visibility*, there is a certain *risk* arising from the physical co-presence with other actors, and there is a *thickness of practice* in digital context stemming from designing for novel functionalities while still maintaining continuity with real-world interaction. However, although integrating several perspectives on embodiment, these themes are meant for general interaction design, not necessarily in learning contexts.

Moreover, often, there is a confusion between embodied learning and embodied interaction [18], with the assumption that designing embodied interaction will necessarily result in embodied learning. However, when considering embodied learning in digital contexts, we need to identify the role of interaction in the learning process. Previous work showed that designing for meaningful embodied interaction, that is, interaction meaningfully grounded in bodily actions [29, 74], does not necessarily result in increased learning outcomes. Interaction-related design choices influence the learning process as they influence the meaning highlighted by the interaction, for example in mathematics [17, 19] or data visualization [7]. In addition, research on movement design demonstrated the importance of separating proximal movement, that is, in our case, bodily actions, from distal movement, that is their actual effect on the world, product of the interaction process [2]. Indeed, in this gap lies the space of sense-making, for example through the creation of attentional anchors, that is imagined and spatially located instruments to support sense-making [2].

Generally, we need to ask whether making sense of the designed interaction supports making sense of the mathematical concept accessed through said interaction. As such, in this work, we ask: How do learners move when making sense of mathematics and how can interaction design support and leverage such bodily actions?

2.2 Individual differences

Embodied activities rely on sensorimotor simulation [59], and therefore their impact depends on how learners perceive and interact with space [53, 94]. As we are interested in bodily actions performed by learners when making sense of mathematics, we focus this section on individual factors relevant in embodied math learning.

2.2.1 Math anxiety. Math anxiety is "a feeling of panic, helplessness, paralysis and mental disorganization that arises when one is required to solve a mathematical problem or manage numbers" [10, 19]. It develops as early as elementary school and persists or increases into adulthood [30, 104]. Women, in particular, report more math anxiety than men [11]. The effect of math anxiety is complex, it may lead to worse mathematical achievement, however, there are also high performing but math-anxious individuals [26, 30, 84, 104]. Math anxiety is a problem in and beyond classrooms. Therefore, teachers and researchers are committed to designing learning environments that help students reduce their

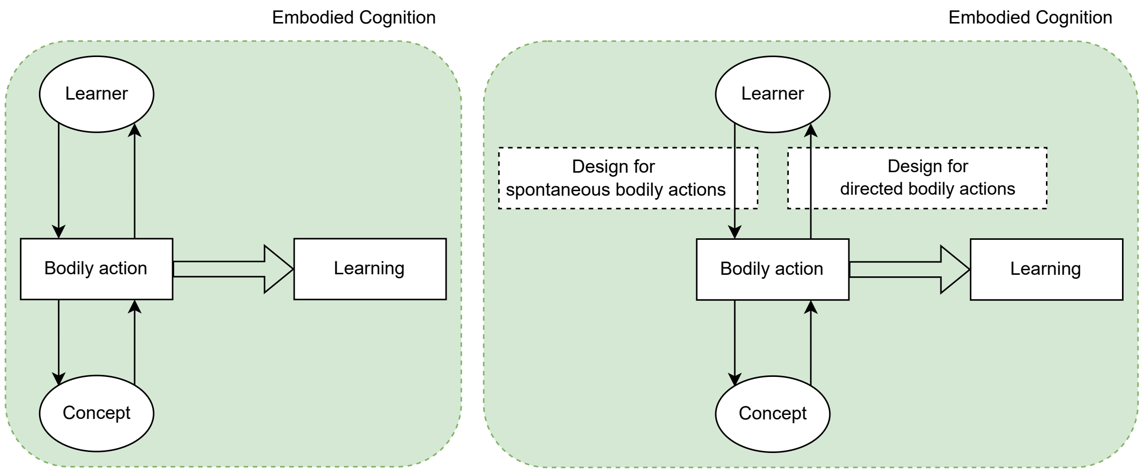

Figure 2: System representation of embodied cognition as defined by Chatain et al. [18] (left), and our adjusted system representation of the design space of embodied cognition (right).

math anxiety, have a positive learning experience, and improve their learning. Specific ways include the use of manipulatives, music, and games [98], or the use of technologies such as Augmented or Virtual Reality [34, 88].

Bodily actions combined with math concepts might support learners' understanding of their bodies in space [83] and allow the learners to make meaningful connections to other areas of life and increases enjoyment and excitement about math [52]. This potentially leads to positive experiences with math learning and reduces math anxiety. Scoop!, a game prototype, explores the integration of movement in a game designed to assist learners with math anxiety, and highlights potential advantages of certain bodily actions to reduce anxiety although more research is still needed [47]. Thus, a playful learning environment encouraging bodily movements may offer a promising approach to decrease math anxiety while increasing interaction and engagement with mathematics.

2.2.2 Body awareness. Body awareness can be defined as an individual's heightened sensitivity to their own physiological processes, that is, one's "attentiveness to normal body processes" [91]. Body awareness is a crucial factor as learners with varying levels of body awareness might rely on distinct sensory cues during their engagement with embodied learning activities. For example, individuals with a strong background in activities where body awareness is heightened, like dance, rely more on proprioceptive feedback than visual feedback [51].

Moreover, body awareness plays a major role in embodied learning activities in VR. In this context, learners interact with digital content through avatars, and the disparity between their physical bodies and their digital representations impacts what is known as the "sense of embodiment" [55]. The sense of embodiment encompasses three key aspects: *self-location* within the digital avatar, *body ownership* towards the digital avatar as an extension of oneself, and *agency* over the digital avatar.

Recent research findings have indicated a notable negative correlation between the degree of individuals' body awareness and their sense of body ownership [19]. This intricate interplay between body awareness and sense of embodiment accentuates the complexity inherent in embodied learning experiences, particularly within the immersive realm of VR.

2.2.3 Mathematical ability. Learners with different math abilities exhibit distinctive movement patterns when making sense of or communicating mathematical concepts. In their study, Gerofsky asked students and teachers to use bodily actions to describe a graphical representation of a mathematical function [37]. They observed that high-achieving students and teachers used coarser movements — that is more expansive, broader movements centered around the waist area, while hard-working students with comparatively lower academic performance exhibited more precise movements, typically at eye level.

Moreover, a study by Nathan et al. [77] revealed that experts harnessed a greater repertoire of representational gestures compared to non-expert, with an emphasis on dynamic representational gestures. Finally, recent work explored whether experts can be identified solely by their gestural expression and revealed that experts produced fewer gestures overall compared to novices. However, their gestural expression was marked by the prevalence of iconic gestures, which contributes an additional dimension to our comprehension of mathematical ability [95].

In summary, mathematical ability plays a multifaceted role in embodied learning, influencing how learners physically engage with mathematical concepts and the distinctive gestural expression they employ to convey their understanding.

2.3 Present work

Embodied approaches have been investigated for various topics of mathematics, such as counting [102], proofs in geometry [78], proportions [45], graph theory [21], and much more [6, 99]. We chose to focus on derivatives as this topic can be related to diverse past embodied experiences, such as slopes and speed; embodied experiences of derivatives are extensively investigated in embodied learning literature, through position-based embodiment, movement-based embodiment, time-based embodiment, or as part of sensorimotor regulation [19, 97, 99]; and, finally, derivatives understanding can be transferred to various fields of study such as physics, engineering, economics, and more.

In this work, we are interested in the design of embodied interaction to support embodied learning of mathematics. We argue that such interaction must not be agnostic to the embodied learning process and should, therefore, acknowledge, support, and leverage the bodily actions learners perform when making sense of mathematical concepts. Learners rely on two kinds of bodily actions: directed bodily actions, and spontaneous bodily actions. To inform future interaction design, we explore both through two studies.

In Study 1, we explore how learners move in an intuition-building activity about derivatives, specifically designed from an embodied learning perspective [19]. To frame our analysis, we focus on individual factors relevant for embodied math learning: math anxiety, body awareness, and mathematical ability. With this, we identify diverse ways in which learners leverage directed bodily actions.

Through a subsequent study, Study 2, we complete our exploration by investigating the various spontaneous bodily actions performed by learners with different math affect and math ability during an intuition-probing task. We identify key characteristics of spontaneous bodily actions that should be considered in the design of embodied learning activities.

We conclude by discussing our results within the general process of embodied learning and offer design recommendations and avenues for future research. This work was approved by ETH Ethics Commission as proposal EK 2021-N-169.

3 STUDY 1

First, we focused on directed bodily actions, that is bodily actions performed because the task requests them [68, 101]. To do so, we designed an embodied learning activity on the topic of derivatives. For a thorough description, evaluation, and validation of our design, please refer to our prior work [19]. In this section, we summarize the activity design and the data collection process. In turn, we describe our analysis and results.

3.1 Embodied Derivatives

Activity. We designed an activity to teach derivatives to high school students through an embodied game [19]. In each level of this activity, a function curve, in yellow, and a corresponding derivative curve, in pink, are displayed. On the derivative curve, a target area is displayed, in pink, with lower opacity. The students are informed that "There is a special link between these curves that you have to discover!", and they can do so by solving puzzles where they manipulate the function curve to fit the derivative curve in the target area (instruction sheets available in the supplementary materials). On the function curve, wooden handles can be manipulated to influence its shape following a cubic spline model [60]. At any point in time, the handle's slope corresponds to the local slope of the curve, and, therefore, the derivative (Figure 3).

The activity was designed to highlight specific learning goals defined according to the high-school program in Switzerland, detailed in Appendix A. Moreover, the activity was implemented as part of a Problem Solving followed by Instruction (PS-I) pedagogical pattern. In this context, the activity served as a Problem-Solving phase and emphasized the three necessary mechanisms [64]: *prior knowledge activation* by reusing curves that the students were familiar with, *deep feature recognition* by focusing the interaction on characteristic points of the function's curve, and *knowledge gap awareness* by including delayed feedback levels. Specifically, the activity contained two types of levels. Normal levels provided an immediate update of the function curve and the derivative curve upon manipulation. Delayed-feedback levels only updated the curves' shapes and positions upon the release of the handle. The different levels are summarized in Appendix A. To avoid fatigue effects, we kept the activity short (under 20 minutes) [19].

Finally, the activity was implemented under two conditions (Figure 3). One condition, TAB, was implemented on a tablet and focused on a lower degree of embodiment [50], with lower sensorimotor simulation and lower immersion. In contrast, a high degree condition, VR (*DIR* in the original paper [19]), was implemented in VR, with high sensorimotor simulation and high immersion.

Data collection. Using this activity, we conducted a user study with high school students. The protocol included two interventions, conducted a few days apart. Both interventions happened at the schools, in a large room, in half classes (groups of about 12 students). Students were positioned so that they could not see the progress of the other students. A thorough description of the setup is presented in Appendix B. The first intervention included a prerequisite test, a demographics questionnaire, a math anxiety questionnaire [44], a body awareness questionnaire [91], and a VR initiation focusing on hand tracking. All participants joined this session, regardless of their assigned condition to avoid disappointment when comparing their experience with their peers before the second intervention. The second intervention included a first baseline Simulator Sickness Questionnaire (SSQ) [54], the activity itself, either TAB or VR based on random assignment, a second SSQ for comparison, a System Usability Scale (SUS) questionnaire [15], a sense of embodiment questionnaire [85] only for participants in the VR condition, an agency questionnaire [40], a short instruction video on derivatives as part of the PS-I pattern, a break, and a post-test including questions from a Calculus Concept Inventory (CCI) [33]. For consistency and to prevent fatigue effects due to differing questionnaire completion times, we used the same questionnaires in both conditions, where applicable, allowing direct comparison between conditions.

All questionnaires used are standardized and validated, except for the prerequisite questionnaire and the learning post-test. These questionnaires were designed according to the high-school program on derivatives in Switerland and validated by three individuals with expertise above master level in mathematics and teaching experience (questionnaires available in the supplementary materials). The design of the questionnaires is discussed in [17]. Cronbach's αs are reported in Appendix C and all questionnaires used in the analysis have "acceptable" to "good" reliability, $\alpha > 0.8$ resp. $\alpha > 0.7$ [38].

Participants. We recruited $n = 149$ high school students. As some students missed the second intervention due to sickness or absence, our final sample size is $n = 130$. In average, the participants were $M = 17.10$ years old ($SD = 0.61$). 63 participants identified as girls, 64 as boys, 0 as non-binary or third gender, and 3 were

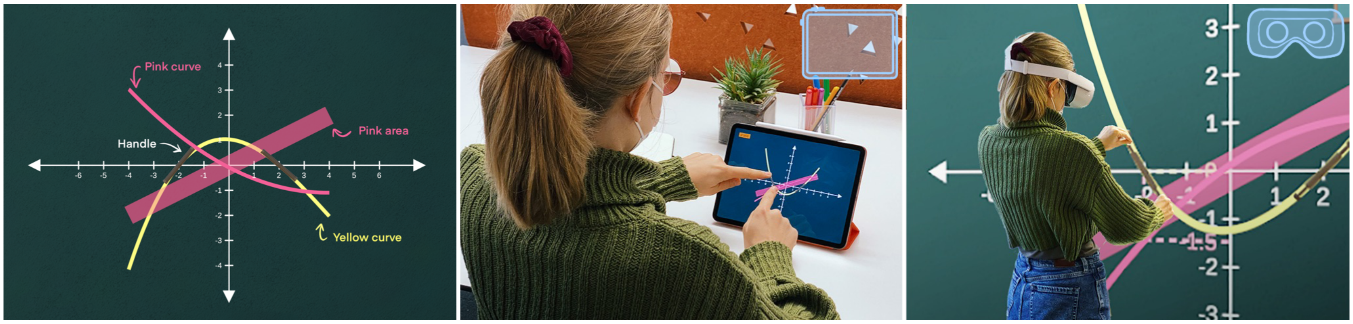

Figure 3: Each level of the activity contains a yellow curve, representing the function, and a pink curve, representing the derivative. The goal is to manipulate the handles on the yellow curve to fit the pink curve into the pink target (Left). This activity was implemented on a tablet (Center) and in VR (Right).

unspecified. All participants were high school students, and the intervention was conducted a few weeks before the lecture on derivatives: the students already studied graphs of functions, but did not study derivatives yet. The participants were split into two conditions: $n_{TAB} = 66$ participants were assigned the lower degree of embodiment condition, on tablet, and $n_{VR} = 64$ participants were assigned the higher degree of embodiment condition, in VR.

3.2 Research question

We performed an exploratory analysis to understand the role of individual factors on how students interact with embodied sense-making activities, focusing on directed bodily actions for derivatives. We aim to gather an in-depth understanding of the behaviors to account for when designing embodied learning activities.

Specifically, we addressed the following research question:

RQ1 How do students move when making sense of derivatives in an embodied activity focused on directed bodily actions?

For clarity, this section focuses on how we analyzed the data, while our results are presented in the next section.

3.2.1 Individual factors. We focused our exploration on individual factors known to impact sense-making of derivatives in embodied learning activities: math anxiety, body awareness, and mathematical ability. These metrics were measured during the first intervention. Math anxiety was measured using the Abbreviated Math Anxiety Scale [44]. Body awareness was measured using the dedicated body awareness questionnaire [91]. We used self-reported math grades as a proxy for mathematical ability.

3.2.2 Preliminary embodied exploration. To perform an initial exploration of the data, we implemented visualizations of the students' actions for each condition. With our system, we could explore the embodied trajectories in space and time for all the students, both on tablet and in VR. We present static examples in Figure 4. This approach, recommended in embodied analysis and fused twins research [41], helped us understand the kinds of movements the students performed and, therefore, define our metrics.

3.2.3 Behavior metrics. First, we considered which information is valuable for our behavior analysis. From the logged data, we had access to the following information:

- Level: Level start, end, and score,
- Interaction: Interaction with handles start, end, and intermediate states including handle state and hands' positions,
- Position: User location in the scene at each point in time.

In this work, we focus on hand movements and decoupled behavior into two aspects: interaction and movement. Interaction is focused on the goal of bodily actions, for example, manipulating a handle. In contrast, movement is focused on the bodily actions themselves, for example, the speed and amplitude of the interaction. We separated these two aspects as previous work shows that manipulation and embodiment lead to different learning outcomes [21].

To evaluate interaction, we considered the number of interactions with each handle, particularly which handles were interacted with, as well as the sequence of these actions. To evaluate movement, we considered the angular amplitude of the movement as well as the sequence of such amplitudes. This is based on previous research highlighting that when communicating about functions' graphs, high-achieving students perform coarse movements while hard-working but low-achieving students perform finer movements [37]. We removed movements smaller than 1 degree as jittery movements due to tracking limitations. We then defined the threshold between small and large movements by using the median of movement amplitudes for each condition, which is 5.41 degrees. Importantly, we separated each interaction into several movements for each change of angular direction. This accounts for students who did not release the handle between different attempts towards the solution.

Finally, for both of these approaches, we also considered long idle states as a proxy for states of reflection. We selected a floor threshold of 5 seconds for such reflection states. This corresponds to the beginning of the right tail of the normal distribution of the base 10 logarithm of duration of idle states. This value is slightly lower than values used in previous work, for example, 8 and 10 seconds [92, 93] because our activity is faster paced than the ones used in these works. A summary of the events used in the sequences is presented in Table 1.

3.2.4 Behavior analysis. For all the analyses, we discarded level 0, the interaction tutorial.

For our numerical metrics, we performed a correlation analysis. When considering number of states (as defined in Table 1), we used Kendall's τ as the data presented a large number of tied ranks.

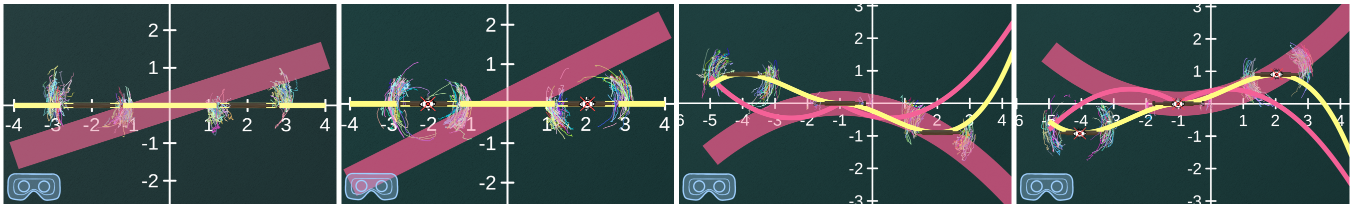

Figure 4: Embodied analysis of students trajectories: Levels 15, 17, 20 and 21 in VR, zoomed in for clarity.

Table 1: Nodes used in the sequence analysis.

Interaction	
INTX	The learner is interacting with the handle in position x.
Movement	
SMALL	The learner’s hands are performing an interactive movement of small amplitude.
LARGE	The learner’s hands are performing an interactive movement of large amplitude.
Other	
REFL	The learner is idle for over 5 seconds, this is used as a proxy for reflection.

When considering numerical values, we used the Pearson’s r. We did not correct for multiple comparisons as we focused on planned comparisons related to our specific research question rather than exploring all possible comparisons.

For our sequential metrics, we performed a sequence analysis focusing on maximum contrasts. For example, to analyze the impact of math anxiety, we performed a sequence analysis with two clusters, one composed of 25 % of students with highest math anxiety, and one composed of 25 % of students with lowest math anxiety. Specifically, we ran a data-driven sequence analysis using the Markov-based approach implemented with the `clickstream` package in R [23, 90, 93]. With this approach, we evaluated the transition matrices between the states described in Table 1, focused on interaction or movement. To evaluate behavior evolution, we looked into four activity levels in particular. We used levels 10 and 11 to evaluate initial behavior as these are the first levels with several handles. Both levels are of similar difficulty, Level 10 includes normal feedback while level 11 includes delayed feedback. To evaluate end-of-game behavior, we considered levels 20 and 21.

3.3 Results

In this section, we present the results of our analysis for each metric of interest, focusing on interaction (Figure 5) and movement (Figure 6). For clarity, we only include relevant results.

3.3.1 Math anxiety. For the TAB condition, we found a positive correlation between math anxiety and the number of interactions ($p < 0.001$, $\tau = 0.29$). This tendency existed for all movements: there was a positive correlation between math anxiety and the number of small movements ($p = 0.002$, $\tau = 0.27$) as well as the number of large movements ($p = 0.018$, $\tau = 0.21$). This means that participants who were more math anxious required more manipulations to reach the solution. In contrast, for the VR condition, we found no significant correlations between math anxiety and interaction, nor between math anxiety and movements.

This difference translated to post-test scores. In the TAB condition, we found a negative correlation between math anxiety and post-test scores ($p = 0.004$, $r = -0.36$). In the VR condition, this correlation was not significant ($p = 0.11$, $r = -0.020$).

In conclusion, we found an effect of math anxiety in the TAB condition but not in the VR condition. Previous research showed that immersive technology could increase motivation for highly math-anxious individuals and positively impact learning [22]. Although we believe that motivation might have played a role in our result, we do not believe it is the main explanation, as we found no difference in the sequence of events between highly math-anxious individuals and their counterparts. In contrast, we believe that our results are due to the fact that, through its immersive properties, VR reduces access to social context cues, and therefore reduces social comparison and provides a safe space for exploration [100]. Indeed, part of math anxiety is due to social comparison [25, 30], and in-VR students could experiment with the content without the fear of being judged by their peers or teacher.

3.3.2 Body awareness. For the TAB condition, we found a positive, but non-significant, correlation between body awareness and number of interactions ($p = 0.052$, $\tau = 0.17$). However, more specifically, we found a significant positive correlation between body awareness and the number of small movements ($p = 0.002$, $\tau = 0.26$). In contrast, for the VR condition, we found no significant correlations between body awareness and interaction nor between body awareness and the number of movements. Moreover, for both conditions, we found no correlation between body awareness and the total amount of movement nor between body awareness and the sense of agency on the mathematical curve.

To understand these results, we used our embodied analysis tool to explore students’ trajectories and observed that trajectories in the TAB condition appear overall less precise (Figure 7). As individuals with higher body awareness, such as dancers, rely more on proprioceptive information than visual feedback [51], we believe that, when in a less precise and low embodied environment, they failed to reach their desired position when using small movements and often needed to re-adjust afterwards. Specifically, highly body-aware individuals suffered more from the unproductive gap between proximal and distal movement [2]. The fact that the number of large movements was not correlated to body awareness supports this theory as large movements are not impacted by precision. Generally,

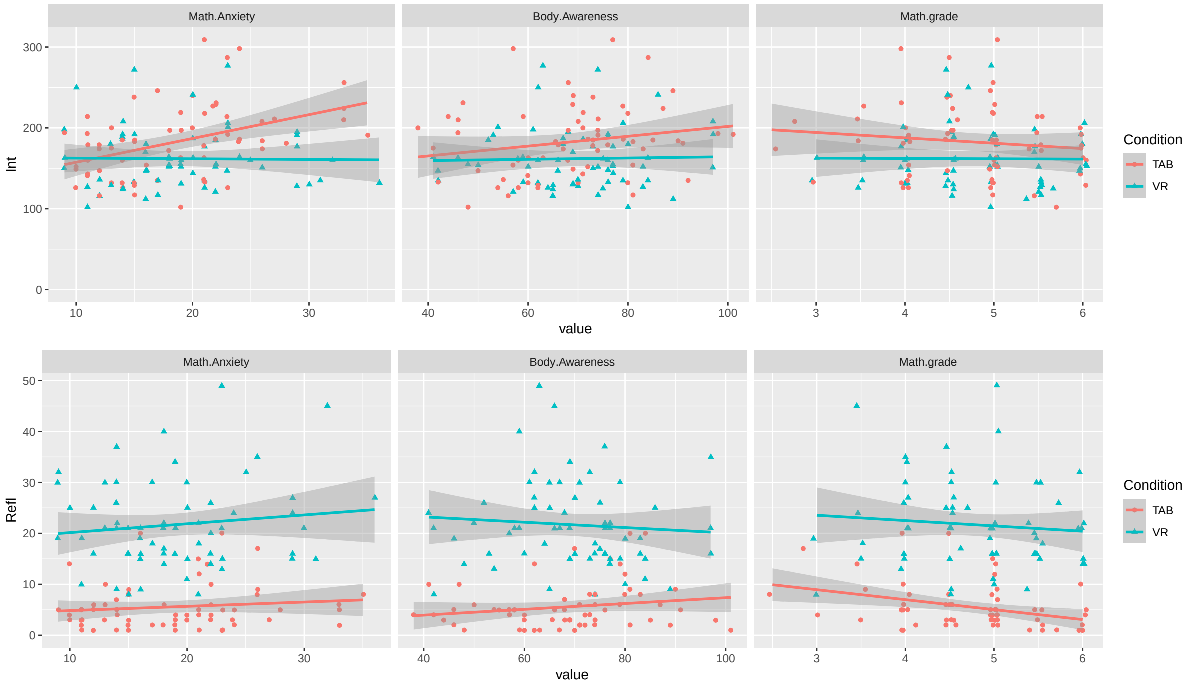

Figure 5: Relation between individual differences and number of interaction events and number of reflection events. The VR condition is represented with blue triangles, the TAB condition with red circles.

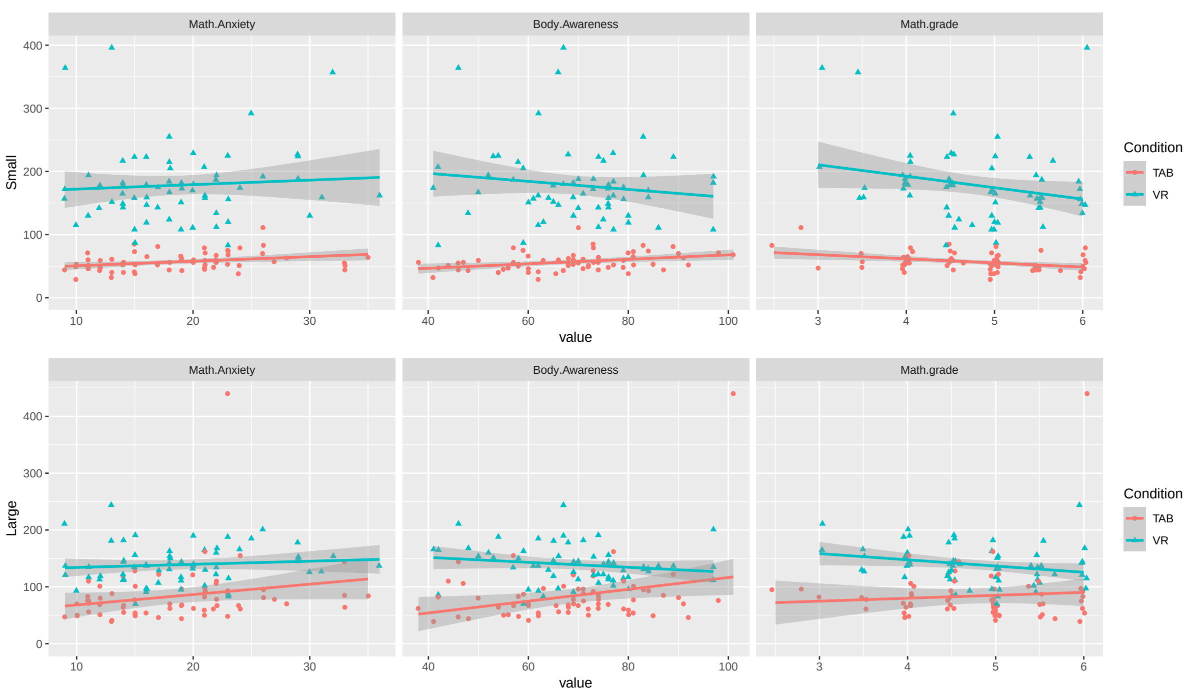

Figure 6: Relation between individual differences and number of movement events. The VR condition is represented with blue triangles, the TAB condition with red circles.

highly body aware individuals may also suffer from the uncanny valley effect of embodied interaction design, as the interaction is close to, yet different from, manipulating a real handle [16].

3.3.3 Mathematical ability. For the TAB condition, we found a negative correlation between the grade of the participants and the number of reflection states ($p = 0.002$, $\tau = -0.23$), meaning that high achieving students needed less states of reflection to reach the solution, or thought less about how to solve the problem and deeper implications. Regarding movement, we found a negative correlation between math grade and the number of small movements ($p = 0.006$, $\tau = -0.25$).

In the VR condition, we found no significant correlation between math grade and overall interaction. However, this lack of correlation might be attributed to the universal engagement of all students with the learning activity, irrespective of their grades. Notably, we found a negative correlation between math grade and the number of small movements ($p = 0.026$, $\tau = -0.20$) and the number of large movements ($p = 0.018$, $\tau = -0.22$). Overall, while the overall number of interactions did not significantly differ based on achievement levels, these interactions were composed of less movements, both small and large, for high achievers compared to lower achievers. This may indicate that considering movement, rather than interaction, is relevant for the design of embodied learning activities.

Through the maximum contrasts sequence analysis, we found a behavior difference between students with highest math grades and lowest math grades, consistent across conditions. At the beginning of the game, students with higher grades followed a REFL ⟶ SMALL transition more often than REFL ⟶ LARGE (54% opposed to 43%). On delayed feedback levels, this tendency was similar although less pronounced (48% REFL ⟶ SMALL, 44% REFL ⟶ LARGE). However, by the end of the game, the tendency was reversed: events of reflection were followed by large movements in most cases (57% REFL ⟶ LARGE as opposed to 43% REFL ⟶ SMALL). This tendency was more pronounced on delayed feedback levels (67% REFL ⟶ LARGE, 33% REFL ⟶ SMALL).

In contrast, students with lowest math grades consistently followed events of reflection by small movements. At the beginning of the game, REFL ⟶ SMALL happened in 67% of cases, as opposed to REFL ⟶ LARGE in 33% of cases. On delayed feedback levels, the tendency was slightly reversed (47% REFL ⟶ SMALL, 53% REFL ⟶ LARGE). At the end of the game, the tendency was the strongest: 72% REFL ⟶ SMALL and 27% REFL ⟶ LARGE following moments of reflection in normal levels, and 64% REFL ⟶ SMALL and 36% REFL ⟶ LARGE on delayed feedback levels.

This illustrates that students with the highest grades had productive moments of reflections towards the end of the game, followed by a large movement to go closer to the solution, and subsequent smaller "fine-tuning" movements to increase the score. This is also illustrated by the fact that large movements were most often followed by small movements (59%). In contrast, students with lowest math grades used small movements after reflection, illustrating that reflection was unproductive and did not support error identification. This is also illustrated by the fact that large movements were most often followed by other large movements (55%).

In conclusion, high achieving students needed less reflection in the TAB condition. In both conditions, high achieving students needed less movements to reach the solution, and in particular less small movements. Moreover, across both conditions, students with higher mathematical ability followed a "think, go to solution, fine-tune" behavior while students with lower grades followed a "think, try, iterate, fine-tune" behavior.

4 STUDY 2

In the previous study, we evaluated the behavior of learners related to directed bodily actions. To complement this work, we now focus on spontaneous bodily actions, that is, we study the bodily actions that participants spontaneously perform when communicating and reflecting about derivatives.

Through this study, we answer the following research question:

RQ2 Which bodily actions do individuals spontaneously perform when making sense of derivatives?

4.1 Demographics

For this qualitative study, we recruited participants with different profiles: novices in mathematics, experts in mathematics, and teachers of mathematics. In Table 2, math level describes the maximum level of education at which the participants used mathematics, math comfort is an average of the self-reported measures of positive math fluency, positive math affect, and negative math anxiety, on 7 points Likert scales. Based on this information, we label P1 and P5 as math novices as they have low math level and math comfort, P2, P3, P4 and P6 as math experts as they have high math level and math comfort, and P3 and P4 as math teachers as they have high teaching experience. There is an overlap between the set of teachers and the set of experts as math teachers are also math experts.

4.2 Protocol

The participants were interviewed online, individually, using Jitsi Meet [1], an open-source video conference software. Due to data-protection concerns, the interviews were recorded locally using the Xbox Game Bar screen-recording feature on Windows.

The intervention included four tasks and a demographics questionnaire, and lasted around 30 minutes. At the beginning of the session, the researcher mentioned that the goal is to answer different questions around four tasks, and that, while communicating their answers, the participants are not allowed to use tools such as their mouse, pen or paper. Gestures were not mentioned explicitly, unless the participants specifically asked. The interviews included four tasks, targeting different scenarios where derivatives are relevant. The three first tasks targeted concrete relatable scenarios, and the last task focused on an abstract curve. For each task, the pictures were displayed on the participant's screen.

Slopes. We first focused on slopes in relatable, visual, and embodied scenarios such as hiking. The users were asked to imagine that they were carrying a heavy backpack and had to describe which path would be most difficult to walk, assuming they start from the red dot and go to the green dot (Figure 8). We then asked them to justify their answer. This task emphasizes derivatives as slopes.

Speed. With this task, we focused on speed, and used the trajectory of a rocket to support this exercise (Figure 9). The users were asked to describe how the speed of the model rocket behaves

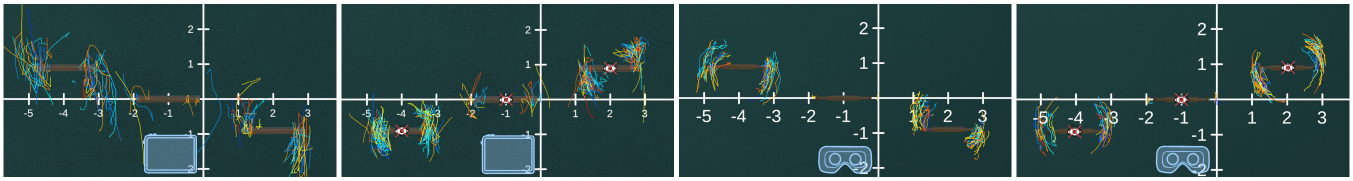

Figure 7: Embodied analysis of high body awareness (reds) and low body awareness (blues). In order, the following are represented: level 20 for TAB, level 21 for TAB, level 20 for VR, and level 21 for VR. For clarity, the pictures are zoomed in, and the curves are not displayed.

Table 2: Demographics of interview participants.

Id	Sitting?	Gender	Age	Math Level	Math Comfort	Teaching	Novice	Expert	Teacher
P1	No	Female	26	Secondary	2.00	No	Yes	No	No
P2	Yes	Male	26	PhD	5.00	Rarely	No	Yes	No
P3	Yes	Male	56	Masters	7.00	Daily	No	Yes	Yes
P4	Yes	Female	22	Masters	7.00	Weekly	No	Yes	Yes
P5	Yes	Female	22	Secondary	1.33	No	Yes	No	No
P6	Yes	Female	19	Bachelors	6.66	Rarely	No	Yes	No

Figure 8: Spontaneous bodily actions: Task 1 focused on derivatives as slopes.

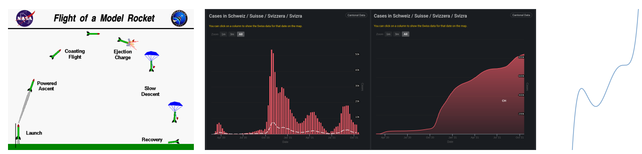

Figure 9: Spontaneous bodily actions: Task 2 (left) focused on derivatives as speed, task 3 (center) focused on derivatives as variation rates, task 4 (right) focused on derivatives as mathematical objects.

throughout the flight from launch until the descent. This task emphasizes derivatives as speed.

Variation rate. With this task, we wanted to move towards abstraction and graphs of functions, while keeping the task relatable. As we looked for graphs that would be relatable for all participants, we decided to use the curves of cases of COVID-19 as a support for the task (Figure 9). We used two graphs: the graph of daily COVID-19 cases in Switzerland, and the cumulative graph of these cases, that is, for each day, the total number of cases until that day. This means that the first graph represents the derivative of the second graph. First, we asked the participants to describe the evolution of daily COVID-19 cases in Switzerland. We then asked them to explain whether the two graphs relate to each other, and how. This task emphasizes derivatives as variation rates.

Abstract. With this final task, we wanted to evaluate a more abstract situation, e.g. the graph of a function as one would encounter in a math class (Figure 9). We selected a function including a local maximum, a local minimum, as well as an inflection point ($f(x) = 0.4x^5 - 1.5x^4 + 4x^2 - 1$). The participants were simply asked to describe the graph. This task emphasizes derivatives as mathematical objects.

4.3 Analysis

To address our research question, we want to identify different behaviors, specifically as expressed through bodily actions, and their relation to the underlying mathematical meaning. As such, we analyzed our data using a reflexive thematic analysis [13, 27], with an inductive orientation to data, and a semantic, experiential and relativist approach [27]. To do so, we first annotated the videos with both speech transcripts and gestures descriptions using the NVivo software [48, 65]. We focused only on the gestures related to derivatives, and ignored body movements such as scratching or moving one's hair. In an iterative process, we then coded our data, using our annotations, but also reviewing the corresponding video extracts for validation. We then defined the themes and focused on patterns of behaviors expressed through bodily actions with shared meaning. This procedure was conducted by the first author, an able-bodied scholar with expertise in mathematics and embodiment research.

4.4 Results

In total, we gathered 111 hand movements, 21 in task 1, 24 in task 2, 29 in task 3 and 37 in task 4. Specifically, P1 performed 24 hand movements, P2 performed 17, P3 performed 3, P4 performed 24, P5 performed 26, P6 performed 17. We believe that the low number of hand movements performed by P3 is due to the fact the P3 struggled to see the pictures on the screen and had to stay close to it in order to answer the questions. In the following, we detail the results of this analysis.

General and specific behaviors are described using different hand poses and trajectories. Hand trajectories were mainly vertical and horizontal, rather than diagonal. Indeed, 35 hand trajectories were vertical and 37 were horizontal, while only 15 were in diagonal. Diagonal trajectories were mostly used by teachers (8 occurrences) and primarily in tasks 4 (6 occurrences) and 1 (4 occurrences). Diagonal trajectories were used in conjunction with a pointing hand pose and used to draw the curve precisely (10 occurrences). In contrast, vertical and horizontal hand trajectories were mostly used with flat or slightly curved hands. Vertical hand trajectories were used in conjunction with words such as "increase", "up", "decrease" or "down", while horizontal trajectories were used with words such as "plateau", "flat", or "stationary". From this, we conclude that learners mostly focus on general up, down, and flat behavior, and do so using flat or slightly curved hand poses. When they need to be more precise, for example to compare different slopes in task 1, or describe a specific shape in task 4, they use a pointing gesture to draw the slope precisely (Figure 10).

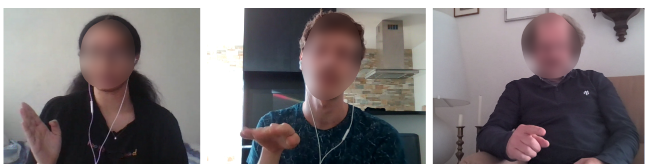

Figure 10: P5 uses a flat hand moved vertically to describe an increase; P2 uses a flat hand moved horizontally to describe a flat point; P3 uses a pointing gesture to describe a specific slope in diagonal.

Steepness is represented with hand tension. Although most gestures are either vertical or horizontal, and the steepness is therefore not always expressed in hand trajectory, we noticed that participants used hand tension to express steepness. By "tension" we do not refer to the mental state of the learner but to muscle tension, meaning "stretched, tight, or rigid". For example, a tensed hand, that is a flat hand, with all fingers connected, is used to represent higher steepness and in conjunction with words like "large", "strong", "sharp" or "very". In contrast, a relaxed hand, that is with slightly curved fingers, a bit spread apart, is used to represent lower steepness and in conjunction with words like "small", "less" or "slight". For example, this was the case to represent mountains as opposed to hills in task 1, or steep variations as opposed to slower variations of COVID-19 cases in task 3 (Figure 11). Sometimes, we observed a mismatch between speech and gesture [86]: For example, on task 3, P1 used words such as "strong" and "very" while describing the later peaks of cases, but used a more and more relaxed hand to support her point. In this case, her gestures described the situation accurately, but her speech was as strong for all the peaks. We believe that this shows that P1 understood the global structure of the graph at the embodied level, but focused on the local perspective at the speech level. From this, we conclude that learners may use hand tension to express steepness, over hand pose and trajectory. We also note that, as described in previous work, a mismatch between gesture and speech should be considered to identify first evidence of learning [86].

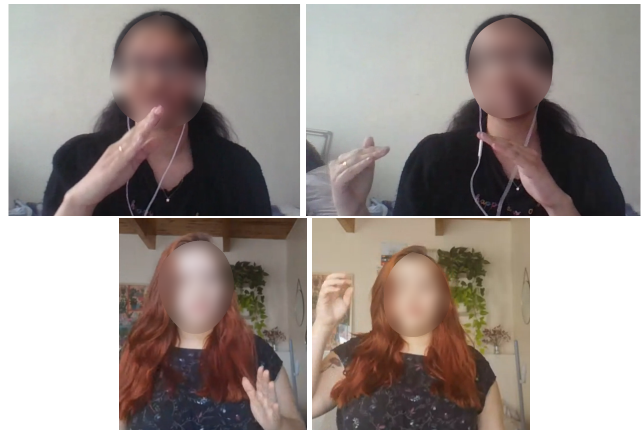

Figure 11: P5 gestures a steep mountain with a tensed hand and a hill with a relaxed hand; P1 gestures a steep increase with a tensed hand and a less steep increase with a more relaxed hand.

Repetition is used to emphasize characteristic behavior. Participants, novices in particular, often used repetition, that is repeating a certain hand movement several times in a row. Specifically, 37 hand gestures used repetition, most often along the horizontal axis (20) and the vertical axis (10). Out of these, 21 were performed by novices. Repetition was most often used to anchor a high level reasoning, either by presenting a characteristic point of a curve or by presenting a key element of the situation. For example, peaks and plateaus were often described using repetition: "The speed stabilizes itself at the top." (P6), "It reaches its peak." (P2), "Then you have plateau." (P4). Moreover, high level reflections are anchored in repetition: "I think there is a bit of distance to walk, it doesn't go up a lot, but it goes for a long time." (P1), "I believe this is why we kept such a low level of corona virus cases." (P6), "There's not many trees or forest around either." (P5). In conclusion, learners might use repetition to emphasize characteristic aspects and deep features of a certain problem or situation. This is particularly important to identify in PS-I pedagogical patterns as recognition of deep features is an important mechanism of learning in this context [64].

Reference points are anchored in space. Some participants used space to anchor reference points and describe a certain behavior in reference to that point (Figure 12). For example, on task 4, P4 first gestured the entire curve, and then started explaining which terms might compose this curve. She hypothesized that, because of the plateau and the high increase at the end, a square term and a cubic term were part of the formula, and positioned them on different points of the curve. In contrast, P6 used explicit anchors for her reasoning. As it is often the case in sign languages, P6 often used both hands, using one as a passive, anchor hand, and one as an active hand. To describe a slope, P6 kept the passive hand at a position of reference, and moved the active hand to draw the slope.

This means that learners' gestures should be considered within their spatio-temporal context.

Figure 12: After gesturing the general shape of the curve, P4 explains where the influence of each term is visible; P6 uses a finger as an anchor to describe a specific slope.

Novices use a first person perspective. Although most participants used a third person perspective, that is looking at the situations as an external observer on the side, P5 often used a first person perspective, that is considering that she was actually standing on the curve (6 out of 26 hand movements). This behavior was consistent across all tasks. Although we did not find evidence of this perspective with P1, the other novice participant, we believe that this is an important aspect to consider. When derivatives are experienced outside of the classroom, at the embodied level, they are often experienced from a first person perspective, for example in a plane or a roller coaster. Although some first person embodied input systems to experience derivatives exist [97], there is no VR activity including this perspective both at the input and visualization level, and we believe this would be an interesting direction to explore in the future.

Embodied metaphors are spontaneously integrated. Participants also used gestures to mimic specific elements of their description in the more concrete tasks (Figure 13). For example, on task 1, participants used their hands to mimic the mountains and hills (P1, P4, P5, P6). Similarly, on task 2, all participants mentioning the parachute also used gestures to represent this parachute during the fall of the rocket (P4, P5, P6). Moreover, on task 3, all participants mentioning a step by step increase also mimicked stairs (P2, P4). This means that, especially when the content is grounded in concreteness, learners might rely on concrete embodied metaphors to express their understanding, rather than directly gesturing the underlying mathematical behavior at a more abstract level.

5 DISCUSSION

This paper aimed to understand how learners use bodily actions in sense-making tasks and draw conclusions on how to design embodied learning activities. To do so, we performed two analyses. In the first analysis, we focused on directed bodily actions in intuition-building tasks on the topic of derivatives. In this context, we explored the variations of bodily actions performed by learners, with a focus of different individual factors. In the second study, we focused on spontaneous bodily actions in a set of intuition-probing tasks on derivatives. This analysis identified key characteristics of bodily actions performed while reflecting and communicating derivatives. We present an overview of our findings in Table 3.

Figure 13: P5 gestures a mountain; P4 gestures stairs to describe a gradual increase; P4 and P6 gesture a parachute.

5.1 Design recommendations

In the following, we elaborate on design recommendations. We followed an iterative process, based on our findings (Table 3), to identify overarching recommendations accounting for both directed and spontaneous bodily actions. Given the exploratory nature of our work, we used a deductive orientation to data for this analysis, contextualized in previous literature (Section 2).

Expand embodied interaction design beyond position and movement. In Study 2, participants relied on muscle tension to express steepness (2b). While previous design frameworks for embodied learning focused on the distinction between position and movement [80] or gestural congruency [50], there is little exploration of muscle tension as a way of expressing mathematical concepts. As such, we recommend exploring interaction leveraging position, movement, but also muscle tension, for example using non-invasive sensors [79]. Considering the direct state induction mechanism [59], we also recommend exploring and leveraging the impact of the felt experience of the learner, considering a *Leib* perspective [74] and a somaesthetic approach to design [42], as muscle tension may be linked to general tension [82].

Consider embodied metaphors and embodied concreteness. In Study 2, participants spontaneously integrated embodied metaphors (2f) and sometimes used first person perspective to represent the concepts (2e). This highlights the importance of considering the role of concreteness in embodied learning as learners spontaneous integrate relatable representations via their bodily actions, but also adopt a relatable perspective. While concreteness has been explored in math learning, and is crucial for learning [61, 76]

Table 3: Overview of our findings, used for the elaboration of our design recommendations.

Study 1 - Directed bodily actions (details in Section 3)	
id	Result ⇒ *Interpretation*
1a	Significant positive correlation between math anxiety and the quantity of movements on tablet but not in VR. ⇒ *Math anxious individuals benefit from embodiment in VR because of the reduced social cues.*
1b	Significant positive correlation between body awareness and the number of small movements on tablet but not in VR. ⇒ *Highly body aware learners benefit from embodied in VR because they rely on proprioception to achieve precise movements.*
1c	First, all performed more small than large movements after reflection states. The trend then reversed for high achieving learners. ⇒ *High achieving learners use a "think and move" strategy while low achieving learners explore until they find the solution.*
Study 2 - Spontaneous bodily actions (details in Section 4)	
id	Summary ⇒ *Theme*
2a	Participants use vertical and horizontal hand movements, using a flat or curved hand, to describe general behavior. For specific behavior, they include diagonal movements and pointing hand poses. ⇒ *General and specific behaviors are described using different hand poses and trajectories.*
2b	Participants used tense fingers for high steepness and relaxed fingers for low steepness. ⇒ *Steepness is represented with hand tension.*
2c	Repetition was used to emphasize key elements, for example peaks and plateaus, or high level reasoning about the task. ⇒ *Repetition is used to emphasize characteristic behavior.*
2d	Participants used spatially anchored elements, either by keeping a hand in one location, or by coming back to that location later. ⇒ *Reference points are anchored in space.*
2e	Although most materials were presented from a side view, a novice participant performed gestures as if she were standing on the curve in a first person perspective. ⇒ *Novices use a first person perspective.*
2f	Participants used gestures to represent elements of the task, such as a mountain or a parachute, but also as metaphors, for example stairs for step by step increase. ⇒ *Embodied metaphors are spontaneously integrated.*

as it supports the modal priming mechanism [21, 59], it often focuses on the representation of the concepts [35], rather than the interaction with said concepts. Therefore, we recommend exploring interaction mechanisms that integrate embodied metaphors [20, 81], and, for VR, different viewpoints [36].

Allow coarse gesturing for identification of deep features. Study 1 revealed that learners with higher mathematical ability performed less small movements, specifically towards the end of the activity where they relied more on coarse movements (1c), and learners with high body awareness performed more small movements in contexts with lower precision (1b). Study 2 highlighted that learners used coarse movement for general behavior, and fine movement for specific behavior (2a). Moreover, learners used repetition of coarse movements to highlight characteristic behavior and deeper reasoning (2c). Overall, coarse movement was used to identify or represent the deep features of a problem or concept, which is crucial for learning [64], and is a sign of mastery [37]. In contrast, fine movement was used to focus on specific behavior or for accuracy purposes. Therefore, we recommend interaction design that allows for coarse gesturing, either by mitigating precision issues, or by refraining from overemphasizing accuracy-based rewards.

Support and evaluate sense-making anchors. Study 1 showed that imprecision is penalizing for highly body-aware learners (1b). We argued this is an unproductive gap between proximal and distal movement [2]. However, such gap can also be productive, for example as a case of desirable difficulty [2, 12, 19], and to support the creation of attentional anchors, i.e. imagined and spatially located instruments to support sense-making [2]. Study 2 revealed that participants relied on spatial anchors to illustrate attentional anchors, for example by using two fingers to draw an imagined slope or relying on previous movements to locate new gestures (2d). Therefore, we recommend giving space for such anchors by considering which interactive elements should be visible, and which should be imagined. For example, in Study 1, the slope is represented as a handle and visible at all times. A similar activity without the handle might create more space for attentional anchors. Moreover, to identify the creation of such anchors, we recommend considering bodily actions performed during reflection times or times of non-interaction, such as pointing and metaphorical gestures, and even consider integrating eye-tracking technology [5].

Integrate embodied in-VR learning assessments. Study 1 revealed that math anxious (1a) and highly body aware learners (1b) benefited more from the embodied activity in VR as opposed to tablet. If we put this in the context of previous literature, arguing that learners are able to express their understanding in gestures, before they are able to articulate it in speech or in writing [86], we question the relevance of disembodied and out-of-VR learning assessments. In alignment with previous recommendations for embodied VR learning activities [49, 99], we recommend considering embodied in-VR learning assessments. To address specific related interaction design considerations, we recommend literature on VR questionnaire design [87] and on mathematics input interfaces in VR [89].

5.2 Limitations and Future Work

There are several limitations to this work. First, the activity used for our directed bodily actions study focused on one interaction technique specifically, the two-hands handle approach, and therefore constrained the exploration space. We believe that this limitation does not invalidate our work as we chose this interaction technique following our previous work on embodied interaction for derivatives [19]. Future work should explore the link between individual factors and bodily actions in other interactive contexts.

Another potential issue is the gamification elements in our directed embodied activity, where a score was displayed on the screen, updating the student in real time on how well they performed [19]. This might have impacted some students as they focused on reaching a 100% score rather than making sense of the problem at hand. While our analysis, using video game experience as an independent variable, found no differences in the dependent variables, we expect that an activity based explicitly on exploration and sense-making may yield different results, and should be explored in the future.

Limitations in the directed bodily actions study include using self-reported math grades as proxy for mathematical ability due to time constraints and ethical concerns. While this might lead to inaccuracies, the self-reporting issue was partially mitigated as many students used their phones to retrieve their grades, but we do not have a way to assess the impact with certainty. Moreover, math grade covers a wide range of topics and could benefit from a more specific assessment. The study's intervention length is another limitation. Indeed, we believe that repeated sense-making sessions over a longer period of time would enable future researchers to evaluate the behavior evolution from novice to expert. Sense-making is a process of several steps [28], and our intervention was too short to cover this progression. This is a general issue in the field of embodiment research [6]. Finally, all participants were recruited from the same area of Switzerland. Although this is a highly international area, we do not believe that our study is sufficient to account for cultural diversity and its impact on bodily actions.

Regarding our spontaneous bodily actions study, the main limitation is the low number of participants as well as the lack of cultural diversity. Although we recruited participants from diverse countries and backgrounds, five participants are not enough to account for all possible perspectives and approaches. However, as part of our exploratory work, we believe that our results already present several avenues for future work and highlight the gap between design for directed bodily actions and design for spontaneous bodily actions. However, we believe that future studies with more participants would help highlight other interesting research directions. For example, future work could investigate the role of attentional anchors in resulting spontaneous gestures in sense-making contexts [2]. Moreover, future work should explore spontaneous gestures production across a panel of users with diverse bodies [53, 94]. Indeed, different individuals move so differently that it is even possible to identify them through hand tracking data [63].

6 CONCLUSION

We explored bodily actions in mathematical sense-making, focusing on derivatives. First, we conducted a quantitative user study evaluating the role of individual differences on directed bodily actions, revealing that VR provides a safe space for math-anxious individuals, that highly body-aware learners are penalized by unproductive gaps between proximal and distal movements, and that students with higher mathematical ability perform fewer movements, relying instead on productive reflection times. Second, we focused on spontaneous bodily actions and performed a qualitative analysis to identify key characteristics of such bodily actions. This study revealed that learners use coarse movements and focus on general behavior when describing derivatives, use hand tension to represent steepness, emphasize characteristic behavior with repetition, anchor reference points in space, may use first person perspective over third person perspective, and integrate embodied metaphors.

From both studies, we devised design recommendations. First, we recommend to expand design beyond position and movement as learners also use muscle tension to express mathematical concepts. Second, consider embodied metaphors and embodied concreteness in design as learners spontaneously integrate relatable metaphors and perspectives when gesturing. Third, allow for coarse gesturing by mitigating precision issues and reducing emphasis on accuracy as such bodily actions support identification of deep features and are a sign of expertise. Fourth, support and evaluate sense-making anchors are these are spontaneously used by learners and are manifestations of imagined learning artifacts. Fifth, integrate embodied in-VR learning assessments as these support math anxious and highly body aware individuals but also offer opportunities to capture evidence of preliminary learning.

With this work, we explored the design space of embodied interaction for embodied learning and offered novel and actionable recommendations with the aim to chart new avenues for future research.

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A ELEMENTS OF THE DERIVATIVES LEARNING ACTIVITY

The learning goals of the activity are presented on Table 4, and the levels are presented in Table 5.

B CLASSROOM SETUP FOR STUDY 1

The study took place in the high-schools directly. We provide a schematic representation of the setup in Figure 14. The exact dimensions and number of tables might not be exact, as it depended in the specific room used. However, the following constraints were always observed:

- In the tablet area, the students were positioned so that they could not see the screens of other students, unless they actively turned around.

Table 4: The learning goals of the derivatives learning game are designed to focus on core concepts at each level of the activity, while emerging concepts serve as criteria for evaluating a deeper understanding.

Core Concepts	
UP	A (strictly) positive derivative reflects a (strictly) increasing function
DOWN	A (strictly) negative derivative reflects a (strictly) decreasing function
FLAT	A null derivative reflects a constant function
SLOPE	There is a link between the local slope of a function and the value of the derivative
VAR	There is a link between the variations of a function and its derivative

Emerging Concepts	
EXTM	At an extremum, the derivative is null
CST	The derivative does not change if the function is shifted by a constant

Table 5: Summary of the levels of the activity.

Levels	# Handles	Feedback	Primary learning goal	Notes
0	1	Normal		Tutorial, Linear function
1-2	1	Normal	UP	Linear function
3	1	Delayed	UP	Linear function
4-5	1	Normal	DOWN	Linear function
6	1	Delayed	DOWN	Linear function
7-8	1	Normal	FLAT	Linear function
9	1	Delayed	FLAT	Linear function
10	3	Normal	UP	Hyperbola, Handle at $x = -3$ is already correct
11	3	Delayed	UP	Hyperbola, Handle at $x = -3$ is already correct
12	3	Normal	DOWN	Hyperbola, Handle at $x = -3$ is already correct
13	3	Delayed	DOWN	Hyperbola, Handle at $x = -3$ is already correct
14-16	2	Normal	VAR	Parabola
17-18	2	Delayed	VAR	Parabola
19-20	3	Normal	SLOPE	Hyperbola, Handle at $x = -1$ is already correct
21	3	Delayed	SLOPE	Hyperbola, Handle at $x = -1$ is already correct

- In the questionnaire area, there was always a free table between two students.
- The questionnaire, tablets, and VR zones for each student were pre-assigned at random.
- The tablets were setup using music stands as these are more affordable than tablet stands.
- VR students are always facing away from the windows as the light reflected on the snow resulted in occasional tracking issues.
- The two experimenters, including the first author as well as a helper, were available for questions and support, but did not interfere beyond this.
- The lecturers were not present in the room, expect shortly at the beginning and at the end to introduce and conclude the study, as they were teaching a class with their other students.

C QUESTIONNAIRE RELIABILITY

Cronbach's αs are reported in Table 6.

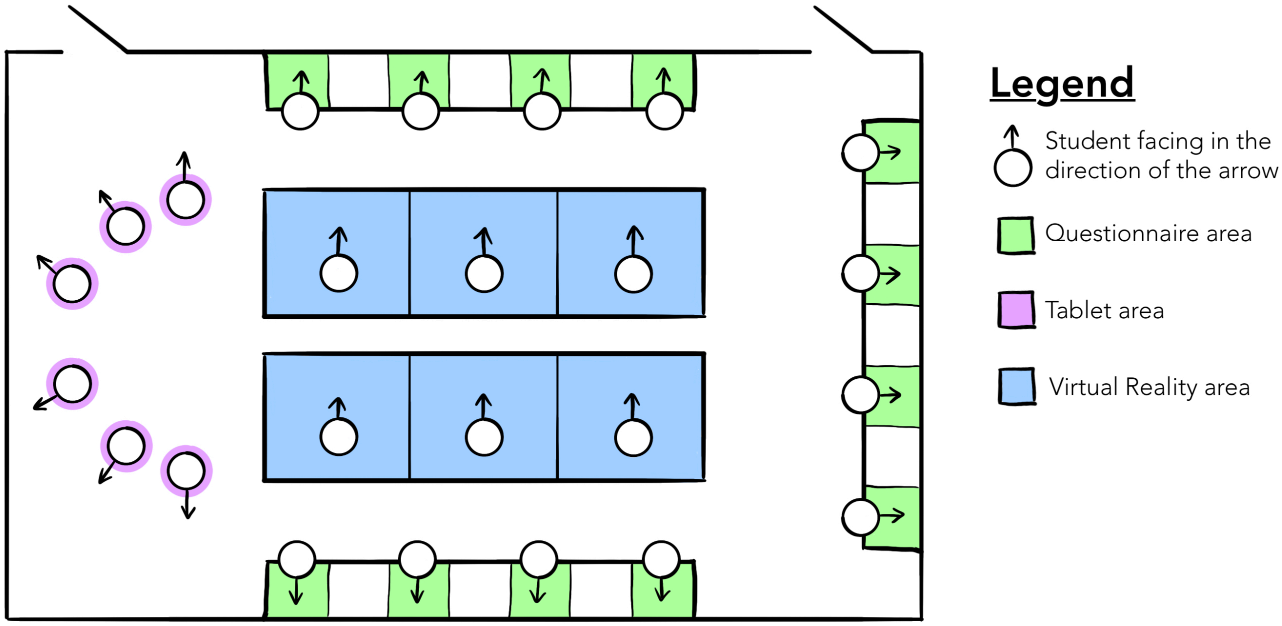

Figure 14: Schematic representation of the study setup in the high school classrooms.

Table 6: Reliability of the questionnaires used in the first study (Section 3). Questionnaires used in the analysis presented in this paper are highlighted in bold.

Questionnaire	TAB Cronbach's α	VR Cronbach's α
Prerequisite test	0.623	0.717
Math Anxiety questionnaire [44]	0.863	0.832
Body awareness questionnaire [91]	0.779	0.760
Baseline SSQ [54]	0.950	0.958
Comparison SSQ [54]	0.953	0.951
SUS questionnaire [15]	0.654	0.790
Sense of embodiment questionnaire [85]	n/a	0.822
Agency questionnaire [40]	0.488	0.404
Learning post-test	0.824	0.893
CCI [33]	0.669	0.502