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Embodied Interaction in Virtual Reality for Grounding Mathematics

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Abstract

Mathematics is a useful skill to learn, even for students who do not wish to become mathematicians. One of the goals of mathematics is to gain understanding of the patterns of the world, in order to model and predict future outcomes. Recent years shed light on the importance of mathematics, for example to understand and take action during the COVID-19 pandemic or the general climate crisis. However, most students find mathematics useless and disconnected from the real world, and several countries are witnessing a worrisome decline in mathematics ability. This means, we argue, that we should challenge our assumptions about how to teach mathematics, specifically as the way we teach mathematics currently does not enable students to understand it well enough to transfer their skills to other classes, nor outside of the classroom setting.

Researchers and educators tackling this issue have argued that, although mathematics is often considered as a Platonic ideal that cannot directly be sensed or manipulated, mathematics rather is a social and malleable process that arises from our sensorimotor experiences of the world. For example, arithmetic can be seen as manipulation of object collections. Similarly, the concept of infinity can be related to our way of expressing something too large for our senses to perceive, while continuity may arise from sequences of elements too small for us to sense. Simply put, mathematics is embodied, and, although not all experts agree on the implications of this theory, there is undeniable evidence that our bodies play an important role in learning mathematics. As a consequence, when designing learning activities, we need to facilitate gesture production, and support sense-making of bodily actions.

Meanwhile, Virtual Reality (VR), a technology heavily focused on bodily movement and manipulations, became more affordable and widespread. Using a wireless Head-Mounted Display (HMD), VR can immerse learners' sensory channels into another world, digitally manufactured. Using hand-tracking technologies, VR seems particularly suitable to support embodied learning activities. With VR, we could create the *MathLand* imagined by Papert, a world of mathematics where learners can explore and manipulate mathematical objects. But is VR truly the solution to support the transition towards embodied learning of mathematics?

Although the idea of using VR to implement embodied learning activities seems

promising, VR interaction research does not leverage actual theories of embodied learning. Specifically, we identified three main challenges in this field. First, there is a lack of empirical studies evaluating the effect of VR embodied learning activities. Second, only a handful of studies address higher education specifically. Third, there are no design guidelines for embodied learning activities in VR focusing specifically on interaction. Considering these challenges, our work addresses the following research question:

How to design embodied interaction to support embodied sense-making of mathematics?

We ground our work in three meanings of embodiment: Embodied cognition relates to the role of learners' bodies in cognition, embodied interaction relates to the role of users' bodies in interaction, and avatar embodiment relates to the perception of a digital body as one's own. Within this framework, we address the challenges with four contributions. Specifically, we address three levels of focus: (1) the avatar level, (2) the interaction level, and (3) the context level. Our last contribution focuses on (4) learners and the bodily actions they perform in sense-making activities.

(1) Looking at embodied interaction at the level of the avatar, we propose "Digital Gloves", a novel input mechanism that supports embodied interaction and reduces split-attention effect by co-locating input and display on the users' hands. Through two user studies, we demonstrate the potential of our mechanism for more intuitive, enjoyable, and effective gaming and learning experiences. We offer recommendations to best design activities using our mechanism as well as suggestions for future applications.

(2) Focusing on the interaction level, we offer an empirical evaluation of the impact of the degree and type of embodiment on usability and learning outcomes. To do so, we designed an embodied activity to learn about derivatives. Our results reveal that although the degree of embodiment only impacts the duration of the activity, the type of embodiment impacts manipulations and learning outcomes. We offer an explanation of these results in terms of mathematical meanings highlighted by different types of embodiment, and conclude with design recommendations for VR embodied learning activities.

(3) In our third project, we focus on the context of the interaction. We conceptualize embodiment as a form of concreteness, and demonstrate the grounding affordances of embodied concreteness. To do so, we designed an embodied activity in VR to teach graph theory to bachelor students. Our activity builds on embodied metaphors by representing graphs as water flow systems. Our results show that students using our activity feel that the activity highlights the relevance of the topic best, and also feel better prepared for the subsequent lecture.

Moreover, unlike an activity focusing on manipulation only, our activity does not impair transfer abilities.

(4) In our last project, we focus on the learner and explore the design space of embodied interaction for sense-making. We explore two contexts. First, we look into an intuition-building activity where learners are directed towards specific bodily actions. Second, we look into an intuition-probing activity where learners spontaneously perform bodily actions. Specifically, we look at the role of individual differences and we aggregate the results from both studies to offer general design recommendations as well as directions for future research on sense-making embodied interaction in VR.

Our work shows that, although VR is a powerful tool to ground abstract mathematics, our interaction design decisions impact how people manipulate the virtual elements and the resulting learning outcomes. Moreover, our work opens a novel avenue of research by highlighting the importance of considering in-VR embodied learning assessments.

Résumé

Il est important d'apprendre les mathématiques, même pour les étudiants qui ne souhaitent pas forcément devenir mathématiciens. Un des buts des mathématiques est de comprendre les schémas qui composent le monde afin de modéliser et prédire le futur. Ces dernières années ont démontré l'importance des mathématiques, par exemple pour la compréhension et la prise de décision lors de la pandémie COVID-19 ou de la crise climatique mondiale. Cela étant dit, de nombreux.ses étudiant.e.s trouvent les mathématiques inutiles et déconnectées du monde réel, et plusieurs pays sont témoins d'un déclin inquiétant du niveau en mathématiques. Selon nous, cela signifie que nous devons remettre en question nos présupposés quant à notre façon d'enseigner les mathématiques, en particulier parce qu'actuellement notre approche ne permet pas aux étudiants de comprendre le contenu suffisamment en profondeur pour utiliser leurs compétences dans d'autres cours ou en dehors de la classe.

Chercheurs et éducateurs essayant de résoudre ce problème ont avancé que, bien que les mathématiques soient souvent considérées comme une idée platonicienne qui ne peut pas être perçue ou manipulée, elles sont en fait un processus social et malléable découlant de nos expériences sensorimotrices du monde. Par exemple, l'arithmétique peut être considérée comme une forme de manipulation de groupes d'objets. De la même façon, le concept d'infini peut être considéré comme notre façon d'exprimer quelque chose de trop large pour être perçu par nos sens, tandis que la continuité peut être l'expression d'une suite d'objets trop petits pour que nous puissions les percevoir. Tout simplement, les mathématiques sont explorées à travers notre corps et bien que tous les experts ne soient pas d'accord quant aux implications de cela, il y a des preuves indéniables que notre corps joue un rôle important dans l'apprentissage des mathématiques. En conséquence, lorsque l'on conçoit des activités d'apprentissage, nous devons faciliter la production de gestes et soutenir l'interprétation de nos mouvements corporels.

En parallèle, la Réalité Virtuelle (RV), une technologie centrée principalement sur les mouvements et manipulations du corps, est devenue plus abordable et répandue. En utilisant un casque d'immersion sans fil, la RV peut emmener les canaux sensoriels des apprenant.e.s dans un autre monde, créé numériquement. En utilisant des méthodes de localisation des mains, la RV semble particulièrement adéquate pour implémenter des activités d'apprentissage centrées sur le corps et

l'expérience. Avec la RV, nous pourrions créer le *MathLand* imaginé par Papert, un monde de mathématiques où les apprenant.e.s peuvent explorer et manipuler des objets mathématiques. Mais est-ce que la RV est vraiment la solution pour soutenir une transition vers un apprentissage des mathématiques corporel ?

Bien que l'idée d'utiliser la RV pour implémenter des activités d'apprentissage corporel semble prometteuse, la recherche en interaction en RV n'utilise pas assez les théories fondamentales de l'apprentissage corporel. En particulier, nous avons identifié trois défis principaux dans ce domaine. Premièrement, il y a un manque d'études empiriques évaluant l'effet d'activités d'apprentissage corporel en RV. Deuxièmement, seulement une poignée d'études se concentre sur l'enseignement supérieur. Troisièmement, il n'y a pas de recommandations pour le design d'activités d'apprentissage corporel en RV qui se concentrent spécifiquement sur l'interaction. En prenant en compte ces défis, notre travail répond à la question de recherche suivante :

Comment concevoir une interaction corporelle pour soutenir l'apprentissage des mathématiques ?

Nous appuyons notre travail sur trois approches : la cognition corporelle signifie que les corps des apprenant.e.s jouent un rôle dans leur cognition, l'interaction corporelle signifie que les corps des utilisateurs.trices jouent un rôle dans l'interaction, et l'incarnation d'avatar qui se concentre sur la perception d'un corps digital comme s'il était à soi. Dans ce cadre, nous proposons quatre contributions. En particulier, nous nous concentrons sur trois niveaux : (1) le niveau de l'avatar, (2) le niveau de l'interaction, et (3) le niveau du contexte. Notre dernière contribution se concentre sur (4) les apprenant.e.s et les mouvements qu'iels effectuent pendant des activités de raisonnement.

(1) En nous concentrant sur l'interaction corporelle au niveau de l'avatar, nous proposons les "Digital Gloves", un nouveau mécanisme qui favorise l'interaction corporelle et réduit l'"effet de l'attention divisée" en co-localisant l'entrée et l'affichage sur les mains de l'utilisateur.trice. À travers deux études, nous démontrons le potentiel de notre mécanisme pour des jeux et des activités d'apprentissage plus intuitifs, agréables et efficaces. Nous offrons des recommandations pour le design de ce genre d'activités en utilisant notre mécanisme, ainsi que des suggestions pour des activités futures.

(2) En nous concentrant sur l'interaction corporelle au niveau de l'interaction elle-même, nous proposons une évaluation empirique de l'impact du degré et du type d'utilisation du corps sur la facilité d'utilisation et l'apprentissage. Pour ce faire, nous avons conçu une activité d'apprentissage corporel sur les dérivées. Nos résultats montrent que, bien que le degré d'utilisation du corps n'impacte que la durée de l'activité, la façon dont le corps est utilisé dans l'interaction impacte les

manipulations effectuées ainsi que l'apprentissage. Nous offrons une explication quant à ces résultats en termes de sens mathématique mis en avant par différents types d'utilisation du corps, et nous concluons par des recommandations pour la conception d'activités d'apprentissage corporel en RV.

(3) Dans notre troisième projet, nous nous concentrons sur le contexte de l'interaction. Nous conceptualisons l'utilisation du corps comme une façon de concrétiser, et démontrons que la concrétisation corporelle permet d'ancrer l'apprentissage. À cet effet, nous avons conçu une activité d'apprentissage corporel en RV pour enseigner la théorie des graphes à des étudiant.e.s en licence. Notre activité s'appuie sur des métaphores liées aux expériences corporelles en représentant des graphes avec un système d'écoulement d'eau. Nos résultats montrent que les étudiants utilisant notre activité ont le sentiment que l'activité met en avant la pertinence du sujet et les prépare mieux pour la leçon qui suit. De plus, en contraste avec une activité se focalisant seulement sur la manipulation, notre activité n'empêche pas les étudiant.e.s de transférer leur savoir dans d'autres situations.

(4) Dans notre dernier projet, nous nous concentrons sur les apprenant.e.s et explorons l'espace de design lié à l'interaction corporelle pour le raisonnement. Nous explorons deux contextes. Premièrement, nous considérons une activité de construction d'intuition où les apprenant.e.s sont amenés par l'activité à effectuer certains mouvements. Deuxièmement, nous considérons une activité de sondage d'intuition où les apprenant.e.s effectuent des mouvements de manière spontanée. En particulier, nous explorons le rôle des différences individuelles et nous agrégeons les résultats des deux études pour offrir des recommandations générales quant au design, mais aussi des directions de recherche sur l'interaction corporelle en RV pour une activité de raisonnement.

Notre travail montre que, bien que la RV soit un outil puissant pour ancrer les mathématiques abstraites, les décisions de design impactent la façon dont les utilisateurs.trices manipulent les éléments virtuels ainsi que leur apprentissage. De plus, notre travail ouvre de nouvelles directions de recherche car il met en avant l'importance d'évaluer l'apprentissage par une approche corporelle et en RV.

Acknowledgments

For Papinou and Blueberry.

With love.

As all things, this thesis is not the work of one, but the work of many.

First and foremost, I would like to thank my family. To my parents: Thank you for encouraging me to follow my passions, and for teaching me resilience. I witnessed the hardships you went through so that I can be here today and am forever grateful for all you have given me. To my siblings: Thank you for your unconditional support, for your advice along the way, for your silly jokes and for your witty minds. I am immensely proud of you, and you inspire me at every step of this path. To my partner: Thank you for taking this journey with me, with love and patience. Thank you for listening to my endless rambling about this work, and for reminding me to slow down once in a while.

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Finally, I would like to thank **you** for reading this work. I hope you will find some of it exciting and inspiring and overall enjoy this journey through the many faces of embodiment.

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Introduction

Figure 1.1: *A student learns mathematics in a virtual world where mathematical objects can be manipulated with embodied interaction.*

1.1 Context

Mathematics is the subject of understanding the patterns of the world [Council, 1989], from which we can form models and predictions to inform our decisions. Mathematics is not only important, it is also necessary. We live in a world of data and information, and, as illustrated during the COVID-19 pandemic, many people do not have the mathematical skills required to make sense of this data. Mathematics is also at the core of all the technological advances of our world, and as Machine Learning keeps growing, having a basic understanding of mathematics also means being able to understand the world we live in. Mathematics is not only for mathematicians, mathematics is for all. However, many countries have seen a worrisome decline in mathematics ability in the recent years. In this light, we ought to wonder: Are we teaching mathematics the right way?

Take a moment to picture yourself learning mathematics. What do you see? For most students, mathematics is learned by sitting quietly at a desk, alone. As a result, many students believe that mathematics has nothing to do with the real world, and has to be learned rather than understood, and is done alone [Schoenfeld, 1992]. Often, students beliefs are not aligned with how mathematicians describe their field. Scholars explain that mathematics is a social process [Resnick, 1988], and that it lies in ideas rather than symbols [Lakoff and Nuñez, 2000].

Experts even argue that mathematics is the result of our sensorimotor experiences with the world [Lakoff and Nuñez, 2000]. For example, arithmetic can be described as object collections, while set theory can be expressed as containers and containees, concepts most human beings are familiar with. Moreover, proponents of embodied cognition claim that our bodies play a major role in learning [Abrahamson and Lindgren, 2014], and that it is through mapping with concrete embodied metaphors that abstract symbols gain meaning [Nathan, 2021]. Embodied approaches have been explored to teach a variety of topics, such as geometry and proofs [Nathan and Walkington, 2017], proportions [Howison et al., 2011], or spatial orientation [Tran et al., 2017].

In turn, Virtual Reality (VR), a technology heavily focused on bodily actions, became more affordable and widespread [Milgram et al., 1995]. Using a wireless Head-Mounted Display (HMD) and hand tracking, learners are immersed in a world where they can manipulate mathematical objects through bodily actions (Figure 1.2).

What if we could teach mathematics differently? What if, using novel technology such as VR, we could offer embodied learning activities for mathematics? In the following, we provide an overview of the landscape of embodiment and a description of how VR can be used to implement such embodied learning activities (Figure 1.2).

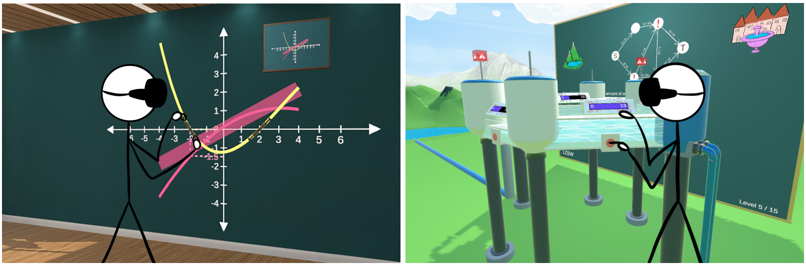


Figure 1.2: *Examples of embodied learning activities in* VR*: "Grasping derivatives" and "Grounding graph theory" [Chatain et al., 2022; Chatain et al., 2023c].*

1.1.1 Embodiment

Generally, embodiment describes the expression of an idea or a concept through a visible or tangible form. However, this term is used in several research fields, carrying different meanings. As this work is anchored both in Computer Science and Learning Sciences, we consider three specific meanings of embodiment.

Embodied cognition refers to the idea that our bodies play a major role in cognition [Abrahamson and Lindgren, 2014]. The implication is dual: learners perform bodily actions while learning, and, in turn, learn from bodily actions. With this meaning, embodiment can be considered from different perspectives: learners' bodies, objects, and environment [Melcer and Isbister, 2016; Ottmar et al., 2019].

Embodied interaction describes the notion that interaction is informed by its social and physical context [Dourish, 2004]. Proponents of the embodied interaction perspective also claim that users' bodies should be considered from the first stages of the design process [Höök et al., 2016]. Moreover, they challenge the traditional approach which considers users' bodies solely as physical entities, and argue that embodied feelings should also be accounted for in design [Mueller et al., 2018].

Avatar embodiment describes the feeling related to experiencing a virtual body as one's own [Kilteni et al., 2012]. This is particularly important when designing VR embodied learning activities as all actions are performed through a virtual avatar. Moreover, avatar representation impacts behavior and performance in VR [Banakou et al., 2018; Kilteni et al., 2013].

1.1.2 Virtual Reality

As opposed to Augmented Reality (AR), an approach including elements of the real world in the experience, VR separates the user from the real world and immerses their sensory channels into the virtual environment.

VR exists in several shapes and forms, based on different hardware such as projectors, mobile phones, or HMDs [Muhanna, 2015]. In this work, we focus on wireless HMD VR with hand tracking instead of controllers. This form of VR is particularly suitable for embodied learning activities: because it is wireless and uses hand-tracking, it supports movement around the space as well as spontaneous and directed hand gestures.

More generally, VR is interesting for embodied learning activities as it enables the creation of experiences that would be difficult of impossible to create in the real world while still supporting for natural interaction [Bricken, 1991; Freina and Ott, 2015]. VR is also suitable because it includes only what we intend to include [Bricken, 1991]: in that sense, it is a controlled environment that we can specifically design to activate precise learning mechanisms. Moreover, the immersion offered by VR, if well integrated in the design, can offer a safe space for students to explore and fail as part of the learning process [Walker et al., 2021; Kapur, 2014].

1.1.3 The Challenges

Although VR is a promising solution to support embodied learning, simply implementing activities in VR is not enough. The design of VR embodied learning activities for mathematics needs to be informed by theory, both from Learning Sciences and Computer Science. However, current research in either field usually does not account for the different meanings of embodiment, and does not leverage findings from the other field. The relationship between different forms of embodiment is rarely considered, and it is often assumed that designing for embodied interaction necessarily fosters embodied cognition.

In this work, we offer an interdisciplinary perspective and are interested in how to design embodied interaction to support embodied sense-making of mathematics.

We identified three main challenges in this field:

- There is a lack of empirical studies evaluating learning outcomes [Ale et al., 2022].

- The potential of embodied learning activities in higher education is under-explored [Tran et al., 2017].
- The design space of such activities lacks guidelines focused on interaction [Abrahamson and Bakker, 2016].

1.1.4 Our Vision

The world we live in requires novel approaches to mathematics teaching and learning, more aligned with advances in the Learning Sciences. Novel technologies such as VR could support this transition, by offering opportunities for embodied learning. However, VR is also cumbersome: it takes more time, space, money and involvement from the teachers. Therefore, as we consider VR to teach mathematics, we ought to make it worth the overheard by offering design recommendations informed by empirical evidence.

With this work, **we offer novel approaches to mathematics teaching, leveraging the potential of VR technology, and informed by theory from an interdisciplinary landscape.**

1.1.5 Thesis Overview

In this thesis, we explore the design space of embodied interaction and how to leverage such interaction to support embodied sense-making of mathematics.

In the next chapter, Chapter 2, we describe the different meanings of embodiment used in this work: embodied cognition, embodied interaction, and avatar embodiment. We then argue that mathematics is embodied, and detail the affordances of VR technology to support embodied learning of mathematics. We conclude by presenting our context for this project, using a system representation based on the three frameworks of embodiment.

In the next chapters, we explore embodied interaction at three levels of focus (Figure 1.3): the avatar performing the interaction, the interaction itself, and the context of the interaction.

In Chapter 3, we focus on the avatar, and present a novel embodied interaction mechanism using the palm of the hand both as input and display. We highlight the potential of this mechanism for gaming and overall playful experiences, as well as for learning.

In Chapter 4, we focus on the interaction, and explore the impact of interaction design on interaction and learning outcomes. Specifically, we look into different degrees of embodiment as well as different types of embodiment.

In Chapter 5, we focus on the context of the interaction, and conceptualize embodiment as a form of concreteness. In this context, we explore the role of concreteness on grounding and learning. To do so, we compare embodied concreteness to a more abstract condition as well as a form of concreteness focused on manipulation only.

We then reconsider the bodily actions underlying interaction by exploring the cycle of embodiment in two directions: spontaneous gesture production, and directed production of bodily actions. In Chapter 6, we explore the bodily actions performed by learners within these two contexts, as well as the role of individual differences.

We conclude by describing implications for future research in Chapter 7.

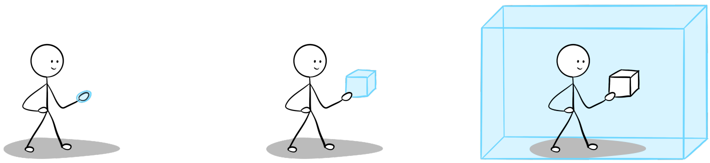


Figure 1.3: *Designing embodied interaction at the level of the avatar performing the interaction, at the level of the interaction itself, and at the level of the interaction's context.*

1.2 Contributions

In this section, we describe the different contributions presented in this thesis. Our three first contributions cover different focus levels: avatar, interaction, and context (Figure 1.3). Moreover, our last contribution focuses on the learners themselves and covers two directions of the embodiment cycle: spontaneous bodily actions and directed bodily actions.

These contributions address the three challenges identified as they are supported by empirical evidence, focus on higher education, and are coupled with design recommendations and implications for theory.

Figure 1.4: *Contribution at the avatar level: A novel input mechanism reducing split-attention by co-locating input and display.*

1.2.1 Avatar Level

In our first project, "Digital Gloves", we focus on the digital avatar and reflect on how to design embodied interaction to bring users' bodies back at the core of the interaction (Figure 1.4). We offer a novel embodied interaction mechanism that reduces split-attention effect by co-locating input and display on the hands of the user. To evaluate our mechanism, we design three embodied activities. In "Space traveller", the game space is directly mapped onto the palm of the user and utilizes the shape and orientation of the hand to provide a novel pinball game experience. By moving their hand, users can transform the shape of the space and the direction of gravity. In the second activity, "Marble runner", the palm acts as a view-port onto the game world, revealing the rest of the level as the users progress into the game. To do so, users can move a marble positioned on their palm by moving their hand around a labyrinth, making sure not to fall. In the final activity, "Noelle's ark", users use both hand to compare different items in weight, acting as a twin-pan balance device.

Through two user studies, we demonstrate the high usability of our prototype and offer design recommendations for future activities using our digital glove mechanism. Although we implemented the activity in VR, we also discuss implications for AR.

CONTRIBUTION
A novel interaction mechanism to support embodied interaction and reduce split-attention effect by co-locating input and display.

Figure 1.5: *Contribution at the interaction level: Empirical evaluation of the impact of the degree and the type of embodiment.*

1.2.2 Interaction Level

In our second project, “Grasping Derivatives”, we focus on the interaction itself and explore the impact of design choices on interaction and learning outcomes (Figure 1.5). To do so, we implement an activity to gain intuition about derivatives. In the activity, the learners need to give a certain shape to a derivative curve by manipulating the function curve. We first evaluate this activity with a panel of experts through a qualitative study. We then evaluate the activity through a quantitative study with high-school students. In this study, we compare different degrees of embodiment, on tablet or in VR, and different types of embodiment, focused on body position or body movement.

We demonstrate that a higher degree of embodiment does not necessarily result in increased learning outcomes. Moreover, a type of embodiment focused on body movement rather than body position can hinder learning and persistence, potentially because it highlights a different mathematical meaning of the derivative that is not congruent with the interaction meaning. We conclude by offering design recommendations for embodied learning activities and discuss the usability, learning, and mathematical implications of such design decisions.

> **CONTRIBUTION**
>
> Empirical evaluation of the impact of the degree and type of embodiment on usability and learning.

1.2.3 Context Level

In our third project, “Grounding Graph Theory”, we highlight the verbal dispute in the field of concreteness in mathematics education, as researchers

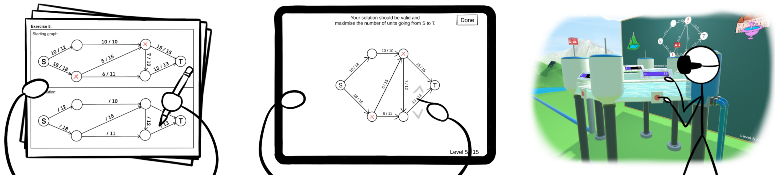


Figure 1.6: *Contribution at the context level: Conceptualization of embodiment as a form of concreteness and demonstration of the affordances of embodied concreteness for grounding graph theory.*

do not agree on the meaning of the words “concrete” and “abstract”. In this context, we focus on the context of the interaction and conceptualize embodiment as a form of concreteness. We implement a VR game to teach graph theory to mathematics bachelor students. In this activity, we represent graphs with a “water flow” embodied metaphor (Figure 1.6). For the students, the goal is to bring the maximum amount of water from a lake to a city by manipulating a pipe system.

After validating our activity through a user study focused on usability, we conduct a quantitative study to demonstrate the grounding affordances of embodied concreteness. We compare three conditions: abstract on paper, manipulated on tablet, and embodied in VR. Through two different measures, our study reveals that embodied concreteness is a powerful tool for grounding abstract mathematics. Moreover, the study shows that, unlike grounding in manipulated concreteness, grounding in embodied concreteness does not impair transfer.

> **CONTRIBUTION**
>
> Conceptualization of embodiment as a form of concreteness.

> **CONTRIBUTION**
>
> Empirical evidence for embodied concreteness as a support for grounding abstract mathematics.

1.2.4 Learner level

In our last project, we explore learners’ bodily actions underlying interaction and, to do so, we take a different perspective on the design space of embod-

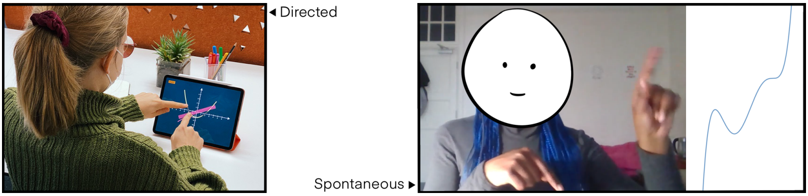


Figure 1.7: *Contribution at the learner level: Exploration directed bodily actions and spontaneous bodily actions.*

ied learning activities. First, we describe embodied cognition as a system where learners learn about a concept through bodily actions. We then argue that this system representation highlights two main directions of importance for design (Figure 1.7). First, learners spontaneously perform bodily actions when learning about a concept, and this implies that embodied learning activities need to support spontaneous movements. Second, learners learn from observing the consequences of their bodily actions, therefore embodied learning activities can direct the learners towards performing certain bodily actions.

In this project, we explore these two directions through two user studies. Moreover, we focus on individual differences of learners to identify how such differences impact bodily actions and how to best support a wide diversity of individuals. Based on our results, we offer design recommendations for embodied interaction, accounting for learners with different levels of math anxiety, body awareness, and math ability.

> **CONTRIBUTION**
>
> Design recommendations for embodied learning activities accounting for both directed bodily actions and spontaneous bodily actions and informed by individual differences.

1.3 Publications, Talks and Service

This thesis is based on the following peer-reviewed publications:

[Chatain et al., 2023a] Chatain, J., Kapur, M., Sumner, R. W. (2023) *Three Perspectives on Embodied Learning in Virtual Reality: Opportunities for Interaction Design.* In CHI EA '23: Extended Abstracts of the 2023 CHI Conference on Human Factors in Computing Systems.

[Chatain et al., 2023b] Chatain, J., Sumner, R. W., Kapur, M. (2023, August) *Embodied interaction in virtual reality for learning mathematics.* In Symposium "A look behind immersive scenes: Experiments on effective learning in virtual reality environments", 20th Biennial EARLI Conference.

[Chatain et al., 2023c] Chatain, J., Varga, R., Fayolle, V., Kapur, M., Sumner, R. W. (2023, February) *Grounding Graph Theory in Embodied Concreteness with Virtual Reality.* In Proceedings of the Seventeenth International Conference on Tangible, Embedded, and Embodied Interaction.

[Chatain et al., 2022] Chatain, J., Ramp, V., Gashaj, V., Fayolle, V., Kapur, M., Sumner, R. W., Magnenat, S. (2022, June) *Grasping Derivatives: Teaching Mathematics through Embodied Interactions using Tablets and Virtual Reality.* In Interaction Design and Children.

[Chatain et al., 2020] Chatain, J., Sisserman, D. M., Reichardt, L., Fayolle, V., Kapur, M., Sumner, R. W., Zünd, F., Bermano, A. H. (2020, November). ***DigiGlo**: Exploring the Palm as an Input and Display Mechanism through Digital Gloves.* In Proceedings of the Annual Symposium on Computer-Human Interaction in Play.

Additionally, during the time period of this thesis, the following peer-reviewed papers were published:

[Grübel et al., 2022] Grübel, J., Thrash, T., Aguilar, L., Gath-Morad, M., Chatain, J., Sumner, R. W., & Hölscher, C., Schinazi, V. R. (2022). *The Hitchhiker's Guide to Fused Twins: A Review of Access to Digital Twins In Situ in Smart Cities.* Remote Sensing.

[Sansonetti et al., 2021] Sansonetti, L., Chatain, J., Caldeira, P., Fayolle, V., Kapur, M., & Sumner, R. W. (2021). *Mathematics Input for Educational Applications in Virtual Reality.* In ICAT-EGVE 2021-International Conference on Artificial Reality and Telexistence and Eurographics Symposium on Virtual Environments. Eurographics Association.

[Chatain et al., 2019] Chatain, J., Bitter, O., Fayolle, V., Sumner, R. W., & Magnenat, S. (2019). *A creative game design and programming app.* In Motion, Interaction and Games.

and the following publications are currently under review:

[Weibel et al., 2023] Weibel, D., Kapur, M., Sumner, R. W., Chatain, J. (2023) *Post Adventures: Co-designing a Computer Science Learning Game with Girls* (Under review).

[Hänni et al., 2023] Hänni, R., Luong, T., Chatain, J., Mangold, F., Dressel, H., Holz, C. (2023) *HistoLab VR - A User Elicitation Study exploring the Potential of VR Game-based Learning for Hazard Awareness* (Under review).

Moreover, the following resources were published:

[Weibel and Chatain, 2022] Weibel, D, & Chatain, J. (2022, July). *Resources for Co-Designing Games with Children.* `https://juliachatain.com/game-design-resources/`

Finally, several other contributions were offered:

- Member at the Communications Committee of the Special Interest Group on Computer–Human Interaction (SIGCHI) 2023.
- Invited Talk "Grounding Abstract Mathematics with Embodied Interaction" at Saarland University, Germany, 2022.
- Panelist "Children & computing: increasing gender diversity" at Interaction Design and Children (IDC) 2022.
- Invited Talk "Grasping Mathematics with Embodied Interaction in Virtual Reality: The Case of Derivatives" at FLI Colloquium, 2022.
- Panelist "IDC for Gender Balance: How can we engage more girls in informatics?" at Interaction Design and Children (IDC) 2021.
- Invited Talk "Grasping Mathematics in Virtual Reality" at Future Learning Initiative Colloquium, 2021.
- Web chair at CHI Play 2021.
- Invited talk "Grounding abstract mathematics through interactive multi-representations" at FLI Colloquium, 2020.
- Invited talk "Reconnecting Mind & Body" at Ludicious "Game Design and Learning Research - How to promote understanding", 2020.

If the reader is interested in citing chapters of this dissertation, several of them can be cited as their corresponding publication directly:

- For Chapter 2, please refer to [Chatain et al., 2023a].
- For Chapter 3, please refer to [Chatain et al., 2020].
- For Chapter 4, please refer to [Chatain et al., 2022].
- For Chapter 5, please refer to [Chatain et al., 2023c].

Theories of Embodiment

The centerpiece of this work is embodiment. The word "embodiment" is used in various fields, with different meanings. Our work is positioned at the intersection between three meanings of embodiment: embodied cognition, embodied interaction, and avatar embodiment. In this section, we provide details on our positioning and describe these different theories of embodiment. As this work is at the intersection between computer science and learning sciences, these meanings belong to either field or span over both of them. Depending on the reader's background, some of the meanings used in this dissertation might be novel. The goal of this section is to clarify these terms and help anchor our work in precise landscapes of research. When describing each perspective, we will focus on the points relevant for grounding our work and conclude by connecting these theories within one system. The caption for such system representation is presented on Figure 2.1. We will then focus on mathematics and the myth of disembodied mathematics. Finally, we will discuss the affordances of Virtual Reality (VR) for embodied learning activities and conclude by summarizing our context and challenges.

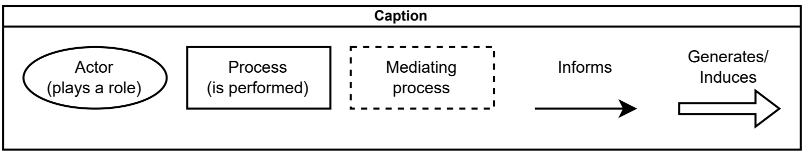


Figure 2.1: *Caption for system representations of embodiment.*

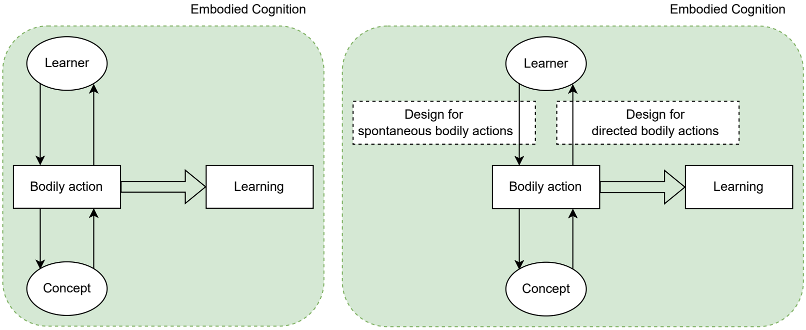


Figure 2.2: *System representation of embodied cognition (left), system representation of the design space of embodied cognition (right).*

2.1 Embodied Cognition

2.1.1 Theory

Although experts do not always align on embodied cognition theory and its implications [Goldinger et al., 2016], there is undeniable evidence that learners' bodies play a role in learning [Kirsh, 2013; Gashaj et al., 2019a; Howison et al., 2011; Nathan and Walkington, 2017]. Specifically, the embodied account of cognition rejects the separation between mind and body, and claims that ignoring learners' bodies is detrimental to learning. While embodied cognition theory has applications beyond our current context, we focus on its role in embodied learning.

The process of embodied cognition goes as follows: When learning about a new concept, learners spontaneously perform bodily actions, and in turn, learn about the concept through their bodily actions. Moreover, research on gestures shows that, when learning a new concept, learners are first able to convey their understanding in gesture, before they can express it in speech and writing [Roth, 2001]. From this perspective, considering thinking or learning without considering the bodies of the learners is a fallacy, or at least, incomplete [Melser, 2004].

Let us consider the following example: a learner is counting the number of apples in her basket. To do so, she extends 3 fingers, 1 finger per apple. Her hand now represents the content of the basket and the quantity 3. In turn, her parent drops another apple in the basket. To account for this change,

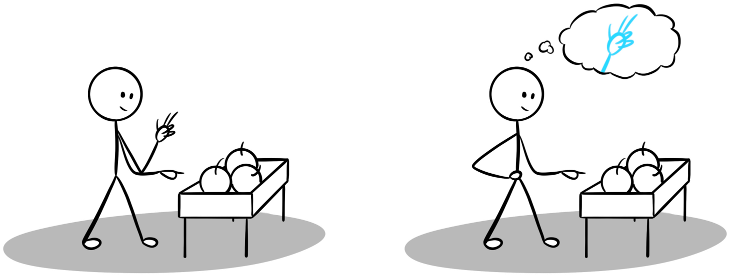


Figure 2.3: *We learn through our embodied experiences of the world [Roth and Jornet, 2013]. As a consequence, thinking is simply a form of truncated action [Abrahamson and Lindgren, 2014].*

our learner extends a new finger, and observes that she now has 4 fingers extended. Through this bodily action, she may learn that $3 + 1 = 4$. This process is represented in Figure 2.2, left.

According to embodied cognition theory, thinking is a form of truncated action. That means that thinking is an internal expression of a physical action truncated before the physical engagement of the body [Abrahamson and Lindgren, 2014]. Concretely, next time this learner will have to count apples, she will plan the execution of the finger counting bodily actions, but not actually externally execute it (Figure 2.3).

More generally, embodied activities support learning in various ways [Pouw et al., 2014; Tran et al., 2017]. For example, in the context of mathematics education, embodiment supports learning: (1) by providing learners a language to reason about mathematical concepts, before introducing symbols and formalisms, (2) by storing information in bodies and objects, and thus alleviating cognitive load, and (3), by making mathematical concepts tangible, and therefore, more concrete. For example, when our learner first figures out how to count, she embodies basic arithmetic using her fingers. She uses this representation to express her preliminary understanding of quantities, simulate basic operations, and store units for further computation.

Conceptually, embodied learning relies on three main mechanisms [Körner et al., 2015]. “Direct state induction” describes the fact that certain bodily states result in certain feelings, independently of higher cognitive processes. “Modal priming” relates to the activation of abstract concepts through sensorimotor states, often via embodied metaphors [Lakoff and Johnson, 2008].

Finally, "sensorimotor simulation" strengthens the link between a stimuli and the simulation of previous bodily actions resulting from this same stimuli.

2.1.2 Opportunities for design

This process of embodied learning can be considered from two main perspectives, with implications for embodied activities design (Figure 2.2, right). First, learners spontaneously perform gestures and body movements while describing and reasoning about mathematical objects. This perspective implies that learning activities and their context should facilitate gesture production. For example, Tancredi et al. designed a balance board input device utilizing learners' need for sensorimotor regulation as part of the learning process itself [Tancredi et al., 2022]. This raises the following question: How can we design embodied activities that enable and support the spontaneous production of bodily action?

Second, learners make sense of concepts by observing bodily actions. Such actions can be produced as directed per an interactive learning activity. For example, in *The Hidden Village*, learners are explicitly taught gestures to represent geometrical concepts and, in turn, perform better proofs in the post-intervention assessment [Nathan and Walkington, 2017; Swart et al., 2020]. In contrast, the *Mathematical Imagery Trainer*, directs learners to move their hands in front of a screen so that it remains green [Howison et al., 2011]. By observing the consequence of their physical movements, students make sense of the concept of proportions. Although none of these bodily actions are spontaneously produced, they serve as anchors for future reasoning. This raises the following question: How can we design embodied activities that support anchoring for embodied cognition and acknowledges individual differences?

As a results, as designers, we can influence the process in two main ways. First, we can design for spontaneous bodily actions, by supporting the path from the learner to the concept. That is, we acknowledge that learners inherently use their bodies as part of their learning process, and design learning activities that enable and support learner's body movements. Second, we can design for directed bodily actions, by supporting the path from the concept to the learner. With this approach, our aim is to direct learners towards bodily actions that are congruent with the concept at hand or explicitly provide learners with a set of bodily actions they can perform to represent certain concepts and support their reasoning.

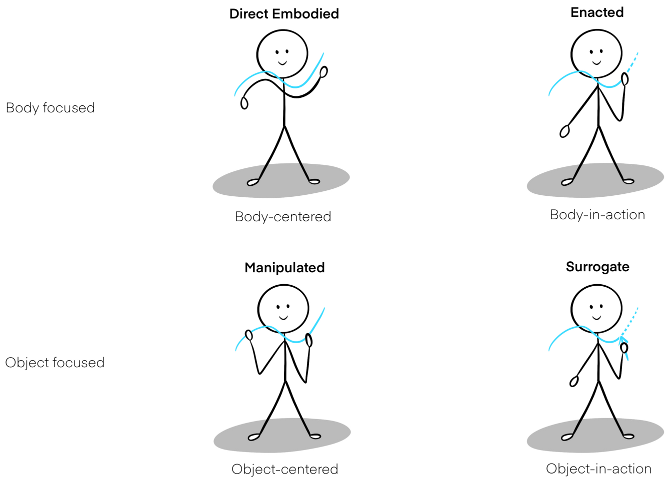


Figure 2.4: *Different types of embodied approaches are used in embodied learning activities [Melcer and Isbister, 2016]. Adapted from [Ottmar et al., 2019].*

2.1.3 Framework

Embodied cognition is often considered as part of the 4E of cognition: Embodied, embedded, extended, and enactive cognition [Newen et al., 2018]. In this work, we consider Melcer's taxonomy of embodied learning, which overlaps with some of these categories, but is more relevant to what we are trying to achieve. Based on this categorization, there are several approaches to embodied learning: body-centered, object-centered, and environment-centered [Melcer and Isbister, 2016; Ottmar et al., 2019]. For the body-centered category, two main approaches can be considered. With the direct embodied approach, one's body is considered as a core component of cognition, and this body's position directly represents a certain concept. With the enacted approach, however, the focus is on bodily action, and the movement of one's body is congruent to the represented concept. A similar consideration can be applied to the object-centered category: in the manipulated approach, an object represents the mathematical object, while in the surrogate approach, the object is used to interact with the mathematical object. These approaches are represented on Figure 2.4.

2.2 Embodied Interaction

2.2.1 Theory

The embodied interaction perspective claims that interaction is informed by and grounded in its physical and social context [Dourish, 2004]. We represent embodied interaction generally on Figure 2.5, and as a system on Figure 2.6.

For example, let us consider the pinching gesture, performed by varying the distance between the thumb and the index finger. This gesture is often used on mobile devices to zoom in and out. This is not a natural gesture, in the sense that this gesture is not used in the non-digital world to zoom on content. However, informed by its physical context, the gesture gains meaning. Indeed, as it is performed on a flat and smooth surface, the fingers metaphorically stretch and compress the underlying digital space.

Dourish also claims that [Dourish, 2004]:

> Embodied interaction is the creation, manipulation, and sharing of meaning through engaged interaction with artifacts.

Here, Dourish emphasizes engaged practice, as opposed to "disembodied rationality", and insists on meaning creation [Dourish, 2004]. In our system representation, the user interacts with an object. This interaction is embodied: it is informed by a social and a physical context. Through this process, meaning-making may occur: that is, the user may make sense of the interaction.

2.2.2 Opportunities for design

Dourish lists several design principles of embodied interaction: the meaning of the interaction should arise on multiple levels; the user, rather than the designer, should create and communicate the meaning; and the interaction should turn the action into meaning. This means that interaction design serves as a scaffold, rather than a guide, to the meaning-making process. Moreover, it is important to note that when Dourish speaks about meaning, he focuses on the meaning of the interaction, of the experience, not necessarily the meaning of a mathematical object.

The idea behind embodied interaction also means that interaction design should be informed by considerations related to users' bodies. There are several ways to achieve embodied interaction design. For example, Höök de-

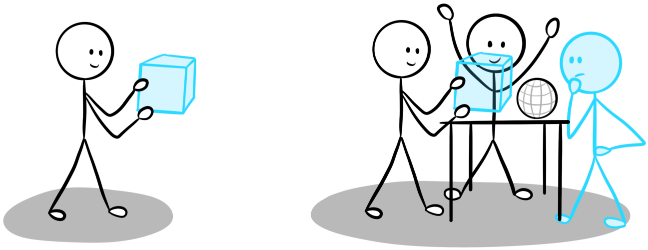


Figure 2.5: *Interaction is embodied and cannot be considered independently of its physical and social context [Dourish, 2004].*

scribes somaesthetic appreciative design, an approach including the users' bodies, and their bodily experiences, from the beginning of the design process [Höök et al., 2016].

Similarly, Mueller et al. present the distinction between the physical body, the *Körper*, and the feeling body, the *Leib*, and argue that embodied interaction design should focus on the *Leib* perspective [Mueller et al., 2018]. When it comes to interaction with digital content, users' bodies are too often considered solely as physical objects utilized to press buttons and perform specific actions with the sole goal of informing the system (*Körper*). In contrast, some body positions and movements are associated with certain feelings and emotions (*Leib*). For example, raising both arms in the air is often associated with feelings of victory, and could be used as such in an embodied activity. As a example, let us consider the design of a "next level" button in a mathematics learning game. Considering the *Körper* perspective, the designer will focus on the physical aspects, and place the button close to the resting position of the hand to avoid tiring the learner. However, considering the *Leib* perspective requires us to empathize with the learner and design the embodied interaction accordingly: At this stage, the learner finally solved this mathematics problem, possibly after several attempts, and learned something new. They feel proud: they accomplished something difficult. As a result, the designer should rather place the button up high, inviting the learner to adopt a "winning pose". Considering the *Leib* perspective is also relevant to activate the direct state induction mechanism of embodied learning [Körner et al., 2015].

Generally, Human-Computer Interaction researchers have been insisting upon the importance of involving users' bodies in the interaction with digital content, although this aspect is still under-theorized [Spiel, 2021].

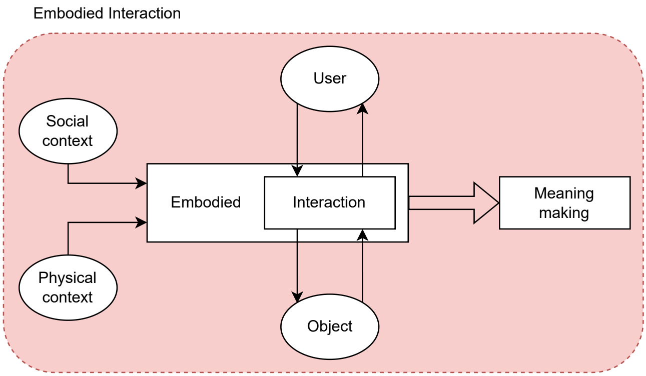


Figure 2.6: *System representation of embodied interaction.*

2.2.3 Framework

When classifying embodied activities, embodiment in a virtual interactive environment can be sorted into different degrees, based on three constructs (Table 2.1): sensorimotor engagement, gestural congruency, and immersion [Johnson-Glenberg and Megowan-Romanowicz, 2017]. Sensorimotor engagement describes the quantity and range in which the users' sensorimotor system is involved. Gestural congruency describes the temporal and spatial relevance of the bodily actions as compared to the underlying represented concept. For example, to input a number, a numerical keyboard offers low gestural congruency while a slider offers high gestural congruency. Finally, immersion, if considered rigorously, describes the objective measure of how much users' senses are immersed in the virtual environment. However, in their definition of immersion, Johnson-Glenberg et al. also include presence, that is the subjective extent to which users feel like they are in the virtual environment and lose track of the real world [Slater, 2003]. VR, specifically, has the potential to support high degrees of embodiment as it offers high levels of sensorimotor engagement and immersion. If designed accordingly, VR activities can also offer high gestural congruency.

Importantly, different degrees of embodiment are often implemented with different technologies. To achieve higher immersion and sensorimotor engagement, VR is a good solution, while lower degrees of embodiment are achieved with screens and tablets. Dourish explains that "Embodiment is not a property of systems, technologies, or artifacts; it is a property of inter-

Table 2.1: *The four degrees of the Embodied Education Taxonomy [Johnson-Glenberg and Megowan-Romanowicz, 2017]. The degrees explored in our work are highlighted.*

Degree	1	2	2	2	3	3	3	4
Sensorimotor engagement	Low	High	Low	Low	Low	High	High	High
Gestural congruency	Low	Low	High	Low	High	Low	High	High
Immersion	Low	Low	Low	High	High	High	Low	High

action" [Dourish, 2004]. However, as these interactions do happen within a technological context, it is important to understand how this impacts the meaning-making capabilities of the interaction. Indeed, there is a trade-off between implementing stronger embodiment with a more cumbersome technology, in particular in a classroom, and implementing weaker embodiment with a technology that is less space- and time-consuming, supports collaboration, and gives a better overview of the task.

2.3 Avatar Embodiment

The definition of avatar embodiment depends on digital avatars, and thus on digital interactive solutions. In this section, we start by defining VR, the technological solution chosen for this work. In turn, we define avatar embodiment and discuss the relevant design space for embodied learning activities.

2.3.1 Virtual Reality

Conceptually, VR is located at the rightmost end of the spectrum between the real environment and the virtual environment [Milgram et al., 1995], as presented on Figure 2.7. As opposed to Augmented Reality that still includes elements of the real environment in the experience, VR completely separates the user from the real world and immerses several of their sensory channels, such as the visual channel and the auditory channel, into the virtual environment. Although this form of VR is often referred to as Immersive Virtual Reality (IVR), we use the acronym VR in this work, for clarity.

Over the years, VR has been implemented in various ways [Muhanna, 2015]. For example, the Cave Automated Virtual Environment (CAVE) is a room-based Virtual Environment, based on the projection of digital content onto

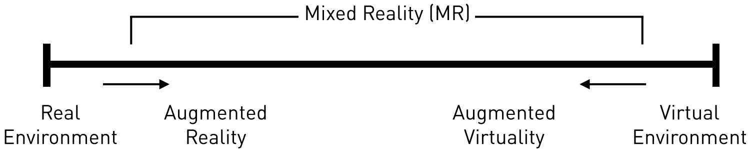


Figure 2.7: *Continuum between the real environment and the virtual environment. Reproduced from [Milgram et al., 1995]. Virtual Reality is at the rightmost end of this continuum.*

the walls, ceiling and floor of the room [Cruz-Neira et al., 1992]. Later, Head-Mounted Display (HMD) became one of the main tools to access VR, in particular as it offers a less cumbersome and less expensive solution [Melzer and Moffitt, 1997]. Still, many hardware solutions are available such as mobile HMDs or wired HMDs [Anthes et al., 2016], and can integrate a wide variety of input signals through hand-tracking [Voigt-Antons et al., 2020], eye-tracking [Clay et al., 2019], physiological sensors [Roo et al., 2017], or haptic feedback [Stone, 2000]. Although VR is often experienced from a first person perspective, other perspectives are also explored [Galvan Debarba et al., 2017]. Finally, in recent examples, VR has also been used for embodied interaction, by integrating the physical context of the interaction [Chatain et al., 2020; Roo et al., 2017], as well as its social context [Marwecki et al., 2018; Wienrich et al., 2018].

In our work, we consider wireless HMD VR experiences, from a first person perspective. As an input technique, we use hand-tracking over controllers, and display digital hands on the real hands of the learners.

2.3.2 Sense of Embodiment

In VR, the visual information received by the user is only digital [Milgram et al., 1995]. In particular, this means that users cannot see their own bodies, and see a virtual avatar instead. Therefore, users manipulate the virtual environment through this virtual avatar. First, the user performs bodily actions, as a puppeteer. These actions inform a digital avatar, which, in turn, interacts with a certain digital object. A summary of this process is presented on Figure 2.8, left.

This process impacts the experience of the users as it influences their proprioception, especially for users with high body awareness [Shields et al., 1989;

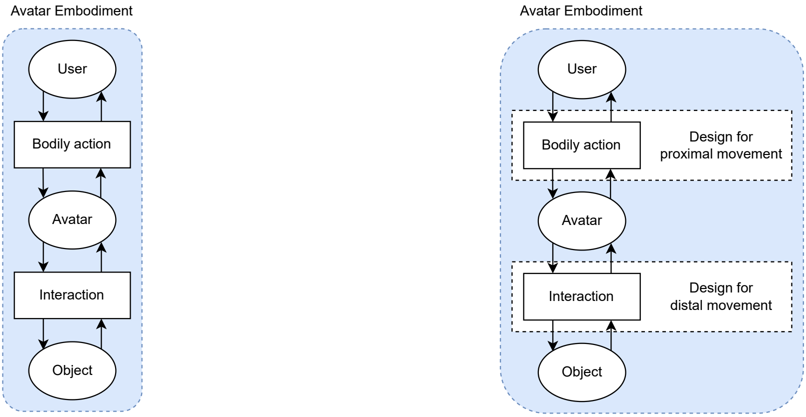


Figure 2.8: *System representation of avatar embodiment (left), system representation of avatar embodiment including design space (right).*

Chatain et al., 2022]. The sense of avatar embodiment describes the connection between the user and the digital avatar, and is rigorously defined as [Kilteni et al., 2012]:

> The Sense of Embodiment towards a body B is the sense that emerges when B's properties are processed as if they were the properties of one's own biological body.

The sense of embodiment is based on three components, schematized on Figure 2.9. The sense of self-location relates to how the digital personal, peripersonal and extrapersonal spaces are perceived in relation to their non-digital counterparts. The sense of agency relates to the sense of being in control of the digital avatar, at a motor level. Finally, the sense of body ownership relates to whether or not users attribute the digital avatars as part of their own bodies.

More recent work defined standardized instruments to measure the sense of avatar embodiment, and focuses on constructs such as the sense of body ownership, the sense of body agency, and the sense of body change [Gonzalez-Franco and Peck, 2018; Peck and Gonzalez-Franco, 2021; Roth and Latoschik, 2019].

To ensure that a high sense of avatar embodiment is achieved, it is recommended to consider the user's perspective in the activity, the sensory consequences of their actions, as well as the morphological similarity between the

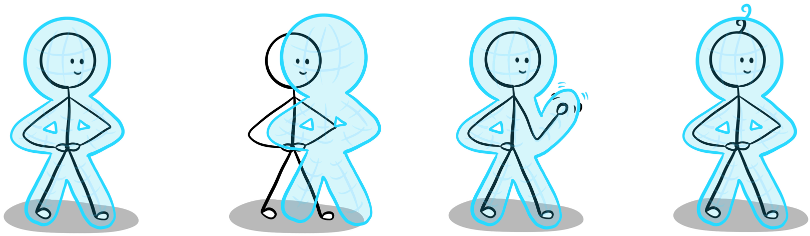


Figure 2.9: *The sense of avatar embodiment in Virtual Reality is composed of the sense of self-location, the sense of agency, and the sense of body ownership [Kilteni et al., 2012].*

digital avatar and the user's body [Kilteni et al., 2012]. In particular, there is an important issue in the field as the diversity of bodies is not sufficiently accounted for [Keehner and Fischer, 2012; Spiel, 2021].

Moreover, the design of the avatar can impact the experience and the abilities of the user. For example, Banakou et al. showed that embodying an Einstein avatar increases male users' performance on cognitive tasks [Banakou et al., 2018]. Similarly, embodying a dark-skinned and casually dressed avatar resulted in behavioral change on a drumming task [Kilteni et al., 2013].

2.3.3 Opportunities for design

Our system representation of avatar embodiment distinguish bodily actions and interaction. This distinction is particularly relevant if connected to embodied cognition. When discussing design of embodied learning activities, Abrahamson et al. underline the importance of the distinction between proximal movement and distal movement [Abrahamson and Bakker, 2016]. When interacting with an instrument, proximal movement describes the bodily actions performed by the learners to interact with said instrument, while distal movement focuses on the actual effect on the world. The gap between proximal and distal movement, they argue, is where sense-making happens. As an example, they describe a circle drawing activity. In one condition, learners simply draw a circle on a touch surface. In a second condition, learners draw a circle using two fingers, one moving along the x axis, and the other along the y axis. In the first condition, there is virtually no gap between proximal and distal movement. However, in the second condition, there is space for sense-making as learners need to understand the construction of a circle within a specific coordinates system to successfully solve the task.

Although, traditionally, the virtual object would be considered as the instrument, we argue that the virtual avatar can also be conceptualized as an instrument, and that, by influencing avatar embodiment, we can design productive gaps between proximal and distal movements. From this perspective, proximal movement describes the bodily actions performed to manipulate the avatar, while distal movement describes the resulting interaction on the world (Figure 2.8, right). We argue that altering the avatar can create gaps between proximal and distal movement, specifically in VR. Previous research in avatar embodiment showed that VR is particularly efficient to support the rubber hand illusion, an illusion that generates a strong sense of body ownership towards a fake hand [IJsselsteijn et al., 2006]. In more recent work, VR was also successfully used to increase body ownership towards a hand with six fingers [Hoyet et al., 2016]. For mathematics learning, this means that one could design an avatar with four fingers per hand to support embodied meaning-making of base 8 counting, or an avatar with stretchable arms to embody 2D transformations in linear algebra, to the point that learners spontaneously generate meaningful gestures that would be irrelevant if performed by their real bodies.

2.4 Embodied Mathematics

In our work, we are particularly interested in how embodied interaction can support sense-making of mathematics. In this section, we describe what we mean by mathematics, and in particular, challenge the idea that mathematics is disembodied.

2.4.1 Mathematics

Before going any further, it is primordial to understand what we mean by "mathematics". In his article "What is mathematics?", Wilkinson offers a list of definitions of mathematics, including but not restricted to:

> Mathematics is the longest continuous human thought; a laboriously constructed intuition; a story that has been written for thousands of years, is always being added to, and might never be finished; the largest coherent artifact that's been built by civilization.

Around the world, mathematics is taught and learned in various ways, focusing on different subsets of the whole field. From geometry to category theory, mathematics also captures a wide range of topics and applications, and can lead one to wonder: What is the underlying essence connecting mathematics together? In their report on mathematics education in the United States of America, the National Research Council wrote [Council, 1989]:

> Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us.

In turn, Pòlya insists that mathematics' main core is sense-making and adds that mathematics is socially constructed and transmitted [Pólya, 1954]. Similarly, Resnick presents mathematics as an ill-structured discipline that students should make sense of, argue about, and create. She also insists on the role that socialization plays in the endeavor: mathematics is more a social process than an instructional process [Resnick, 1988]. In conclusion, mathematics is not necessarily about numbers and symbols, it is about identifying patterns and making sense of them through a social process. It is something to discuss rather than to absorb and digest. Consequently, we can see mathematical language and aesthetic more as tools to communicate and externalize understanding, rather than solely tools to reason with [Lakoff and Nuñez, 2000]:

> The intellectual content of mathematics lies in its ideas, not in the symbols themselves.

Just as one can reason about algorithms without knowing C++ syntax, one can learn to think mathematically before knowing how to communicate with commonly accepted mathematical symbols.

However, when asked how they perceive mathematics, students proved to hold several ideas about mathematics that do not reflect this definition. For example, students believe that mathematics problems have one single correct answer, that mathematics is a solitary activity, and that is has more to do with memorizing than understanding unless one is an extraordinary student [Schoenfeld, 1992]. In the next sections, we list some mechanisms that are important for mathematics learning and are often lacking in institutional pedagogy, resulting in such students beliefs.

2.4.2 The Myth of Disembodied Mathematics

Following Plato's view, mathematics is often considered as a perfect ideal that we, mere humans, can only appreciate through its shadow or projection. In contrast, more recent accounts claim that mathematics comes from our embodied experiences of the world, and is grounded in “situated, spatial-dynamical, and somatic phenomenology” [Abrahamson and Lindgren, 2014; Lakoff and Nuñez, 2000]. For example, one can draw a mapping between object collections and arithmetic, where numbers are the sizes of object collections, and addition describes the action of putting object collections together.

If we accept the embodied account, it follows that our sensory experiences impact mathematics. For example, the mathematical concept of infinity could be explained by the limitation of our senses to comprehend large scale objects such as our planet. Similarly, the concept of continuity could be due to our sensory inability to perceive small scale objects such as molecules. In these terms, we experience infinity and continuity on a daily basis, and can therefore relate with abstract concepts.

There is evidence that we, as humans, embody mathematical concepts. For example, there is evidence that learners subjectively place numbers on an imagined number line, with lower numbers on the left and higher numbers on the right [Dehaene, 1992]. Moreover, number processing activates areas of the brain that are usually activated during sensorimotor simulation [Kiefer and Trumpp, 2012]. Going even further, there is evidence that the way we learn finger counting as children still has an impact on our number processing abilities as adults [Domahs et al., 2010].

Embodiment has also been explored in the design of math learning activities. We detail these approaches through several examples. The *Mathematical Imagery Trainer* is a system where learners are invited to move their hands in front of a screen until the screen turns green. The screen is programmed to turn green when the height ratio between the hands equals a specific predefined value. Once the students successfully manage to turn the screen green, they explore how they can move their hands to keep the screen green. Through this embodied activity, students make sense of the concept of proportions [Howison et al., 2011]. Different flavors of embodiment have been evaluated through empirical studies. In *The Hidden Village*, Nathan et al. showed how teaching physical gestures to students can help them ground their mathematical proofs and support a greater conceptual understanding [Nathan and Walkington, 2017]. Fischer et al. demonstrated that full-body movement can significantly improve learning during numerical training [Fischer et al., 2015]. Petrick et al. showed that over various embodied activities, students in the embodied cognition displayed higher gain in conceptual understanding and provided more detailed answers to the test questions [Petrick, 2012]. Studies and reviews of physical representations also show that, if the representation is congruent with the topic at hand, it can improve the learning rate [Rau, 2020].

These examples are part of a greater trend demonstrating the relevance of motor-action inquiry problems for mathematics education [Abrahamson and Sánchez-García, 2016], and the general importance of manipulating rather than solely observing for learning [Kirsh, 2013]. Overall, embodied cognition approaches for learning benefit the learner by providing a new level to express and reason about mathematical concepts, by grounding cognition in the physical world and thus reducing cognitive load, and by connecting the abstract to the concrete [Pouw et al., 2014; Tran et al., 2017; Kirsh, 2013]. Moreover, studies have shown a developmental link between motor skills and mathematical abilities [Gashaj et al., 2019b; Gashaj and Trninic, 2022], indicating a deeper importance of learners' bodies for mathematics. However, in traditional education, learners have little or no opportunities to manipulate and embody abstract mathematical concepts. Moreover, there are few studies exploring the benefits of physical-based learning for higher education or more advanced topics like abstract mathematics.

In this work, we accept that mathematics is embodied and that embodied activities can support learning.

2.5 Affordances of Technology

Generally, technology has the potential to support learning by providing a space where one can learn by doing, integrating meaningful situated feedback, offering visualizations of concepts otherwise difficult to understand and providing different perspectives on a situation [Bransford et al., 2000]. Researchers focusing on mathematics education have also identified the potential of technology for their field. For example, Artigue explains that, if carefully designed, computer technology can greatly improve cognitive flexibility between different representations of one mathematical concept, but that to this day, this has not been explored enough [Artigue, 2009]. More recent work shows that technology-enabled interactive dynamic visualizations can help students understand the concept of functions better than static images [Rolfes et al., 2020]. Unfortunately, many papers published from a technology-focused perspective do not include a thorough evaluation of the effects on the users [Marek, 2019; Ahmetovic et al., 2019; Ghisio et al., 2017].

As explained, our work focuses on embodied interaction in VR specifically.

In this section, we justify our decision by describing the affordances of VR for embodied learning activities.

VR supports both spontaneous and directed bodily actions VR has the potential to offer the highest degree of embodiment as it offers high immersion and high sensorimotor simulation [Johnson-Glenberg and Megowan-Romanowicz, 2017]. Moreover, VR coupled with hand-tracking is also particularly well suited to increase the sense of embodiment of the user as it can support the sense of self-location, the sense of agency, and the sense of body-ownership [Kilteni et al., 2012]. Finally, VR setups such as wireless standing VR offer a wide freedom of movements, and thus support spontaneous bodily actions. Combined with hand or full-body tracking, such setups also support directed bodily actions. Both of these setups are now widely available. Moreover, as described before, by manipulating the virtual avatar, VR can be used to generate a high sense of avatar embodiment towards a virtual body that is better suited to apprehend certain concepts, such as an avatar with eight fingers to support counting in base 8.

VR grounds in concreteness Abstract mathematics symbols are meaningless unless grounded in concreteness [Harnad, 1990; Glenberg et al., 2012]. VR supports grounding in concreteness, as it enables access to embodied schemata, and offers concrete experiences for students to connect future learning experiences to [Chatain et al., 2023c]. This process is achieved through feedback mechanisms, such as error identification, error understanding, strategy acquisition [Mory, 2013; Fyfe et al., 2012; Wisniewski et al., 2020], and embodiment mechanisms, such as direct state induction, modal priming, sensorimotor simulation [Körner et al., 2015]. Moreover, making use of embodied interaction, VR can create a familiarity with the mathematical objects, another form of concreteness [Dourish, 2004; Dewey, 1910].

VR expands the possible space of exploration Oftentimes, building an embodied activity in a real environment would be expensive, cumbersome, dangerous or altogether impossible. Therefore VR is a particularly suitable alternative as it has the unique potential to provide immersive embodied 3D experiences that could not be accessed in a natural environment while still offering potential for natural interaction [Bricken, 1991; Freina and Ott, 2015]. With VR, learners can interact with elements that cannot usually be sensed, such as mathematical graphs, atoms, or the universe [Chatain et al., 2023c; Edwards et al., 2019; Eryanto and Prestiliano, 2017]. Moreover, in

VR learners can perform experiments that could be dangerous in the real world [Pirker et al., 2017a].

VR provides a safe space for exploration As VR disconnects the learner from the real environment, it enables them to experiment with the learning content without fear of harm, failure or judgment [Walker et al., 2021]. This is primordial as, when carefully scaffolded, experimentation and failure plays a crucial role in mathematics learning [Kapur and Bielaczyc, 2012; Kapur, 2014]. VR is also interesting as it includes only what we intend to include [Bricken, 1991]: in that sense, it is a controlled environment that we can specifically design to follow specific pedagogical patterns and activate precise learning mechanisms.

VR includes playfulness and gamification As most technological learning solutions, VR supports gamification and playfulness. However, by its high degree of embodiment, VR also supports play at the embodied level [Mueller et al., 2018; Chatain et al., 2020]. Concretely, with VR, the learner can reach pleasant movement-induced states through playful interaction with the activity, for example by raising both arms in a winning position when finishing a level [Chatain et al., 2023c].

VR enables embodied learning assessments As learners progress towards understanding a new concept, they are first able to express it in gestures, then in speech, and finally in writing [Roth, 2001]. Moreover, gestures play a moderating role in the effect of directed actions: specifically, students only benefit from these actions if they actually perform gestures during the learning assessment [Walkington et al., 2022]. However, most learning assessments focus on writing, and sometimes speech. This approach is limited as it does not capture the first step of understanding: gestures. Although this has not been explored yet, VR, supported by hand-tracking, has the potential of offering embodied assessments of learning and therefore improve evaluation of learning as well as our understanding of the role of embodiment in this process.

2.6 Our context

For this work, we operate within the system described by three approaches to embodiment: embodied cognition, embodied interaction, avatar embodiment. This system is represented in Figure 2.10. To simplify the figure, we omit certain relationships. For example, in object-centered and enacted approaches to embodiment [Melcer and Isbister, 2016], a large arrow from `Interaction` to `Learning` could be represented. However, we argue that even with such approaches the focus is on bodily actions for learning, although directed by interaction with a certain object, rather than the interaction itself. Moreover, all arrows should be considered as conditional, e.g. a certain process may happen, but does not necessarily.

We believe our interdisciplinary approach is particularly interesting as even though these three aspects play a major role in VR embodied learning, they are rarely acknowledged. For example, recent reviews of embodiment in digital solutions consider either one or two of these perspectives, but never all three aspects [Ale et al., 2022; Duarte et al., 2022; McGowin et al., 2022]. Moreover, to the extent of our knowledge, no research explicitly discusses the gap between these perspectives nor the implications of such interdisciplinary approach on interaction design.

Within this context, we address the following research question:

RQ How to design embodied interaction to support embodied sense-making of mathematics?

In the following, we use the term "embodiment" to describe "embodied interaction to support embodied sense-making".

Although embodiment plays a role in mathematics sense-making, moving does not necessarily mean learning [Tran et al., 2017], and this field suffers from a strong lack of empirical studies specifically measuring learning [Ale et al., 2022]. For example, in 11 years of research in the field for Children-Computer Interaction, only 12% of publications on embodiment addressed learning outcomes [Ale et al., 2022]. Moreover, the rare available quantitative studies fail to show significant learning outcomes of embodied learning activities over their counterparts, and often highlight design limitations [Ale et al., 2022; Mora-Guiard and Pares, 2014; Malinverni et al., 2012]. We hypothesize that these results are due to a lack of interaction-focused design guidelines.

In consequence, we address three challenges with this work: First, there is a lack of empirical studies in the field of embodiment. Second, there is little research on embodiment for higher education [Tran et al., 2017]. Current research targets topics such as proportions [Howison et al., 2011], geometry [Nathan and Walkington, 2017], or spatial orientation [Johnson-Glenberg et al., 2014]. Third, there is a lack of interaction-centered design guidelines for embodiment, and this gap has been identified before [Abrahamson and Bakker, 2016].

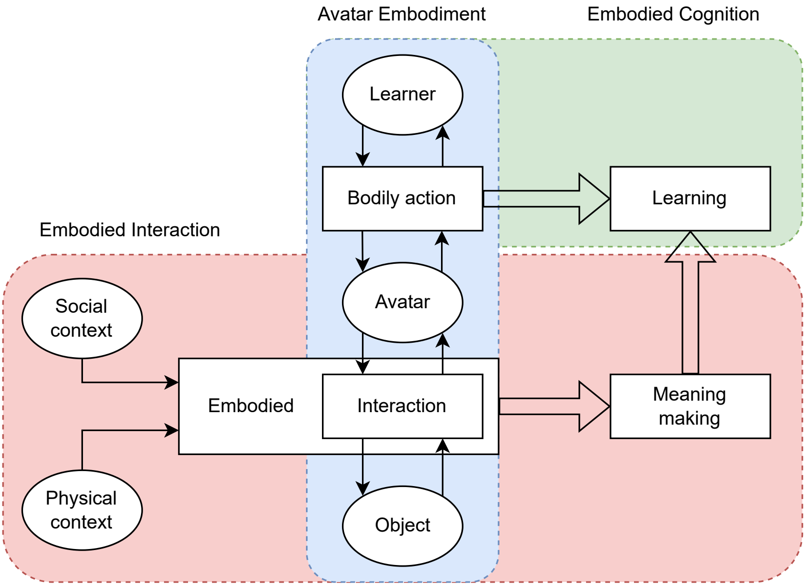


Figure 2.10: *System representation of the embodiment landscape this thesis operates in.*

Johnson-Glenberg et al. identified nine necessary design principles of embodied learning activities [Johnson-Glenberg, 2019]:

- "Scaffold cognitive effort (and the interface)—one step at a time
- Use guided exploration
- Give immediate, actionable feedback
- Playtest often—with correct group
- Build in opportunities for reflection
- Use the hand controls for active, body-based learning
- Integrate gestures that map to the content to be learned
- Gestures are worth the time and extra expense—they promote learning, agency, and attenuate simulator sickness
- Embed gesture as a creative form of assessment, both during and after the lesson."

However, the existing guidelines say very little about how to design the interaction itself. In this work, we explore embodied interaction design at three levels: avatar level, interaction level, and context level. To address the avatar level, we contribute with a novel embodied interaction mechanism that reduces split-attention effect by co-locating input and display. To address the interaction level, we compare different degrees and types of embodiment in a sense-making activity about derivatives. To address the context level, we conceptualize embodiment as a form of concreteness and demonstrate the affordances of embodied concreteness for grounding graph theory. We conclude by going back to the learners and their bodily actions and offering an exploratory account of the design space driven by directed and spontaneous bodily actions. Throughout these projects, we support our work with empirical studies as well as studies in higher education to close the gap in the literature.

Designing the digital avatar

*The shape and function of the human hand are intimately linked to our interaction with the physical world and sets us apart from our evolutionary ancestors. In the digital age, our hands still represent our main form of interaction. We use our hands to operate keyboards, mice, trackpads, and game controllers. This modality, however, separates content display and interaction. In this chapter, we present Digital Gloves (**DigiGlo**), a system designed to evaluate the benefits of a unified hand display and interaction system. We explore this symbiosis in the context of gaming and learning activities where users control elements using hand gestures while the content is displayed on their bare hand. Building on established learning principles, we explore different hand gestures and other specially tailored interactions, through three carefully designed activities and two user studies. From these we show that this is an idea that has the potential to bring more intuitive, enjoyable, and effective gaming and learning experiences, and offer recommendations regarding how to better design such systems.*

Figure 3.1: Space Traveller, Marble Runner, Noelle's Ark*: Three activities that use the palm of the hand as both input and display.*

3.1 Introduction

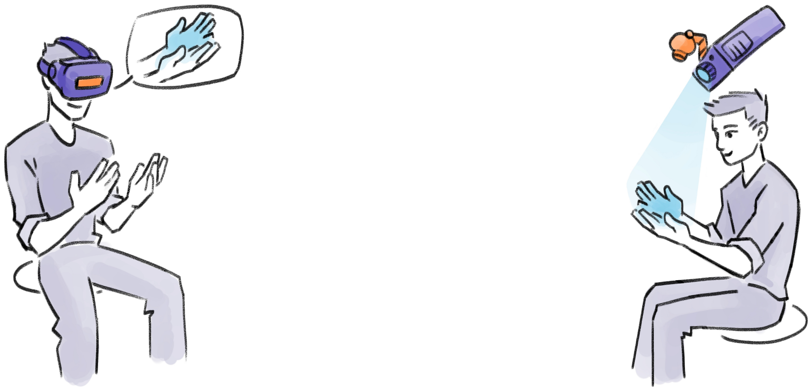
At the core of our embodiment system is the digital avatar. This component, although primordial, is often under-explored in embodied interaction design. In this chapter, we push the boundaries of avatar design and explore how the digital avatar can be used to bring the user's body back at the core of the digital activity. Specifically, we address the following research question:

RQ How can one design the digital avatar to support embodied interaction for playful and learning activities?

Generally, Augmented Reality (AR) and Virtual Reality (VR) have received much interest in recent years, experiencing significant advancements [Brigham, 2017]. They have been successfully employed for a wide variety of entertainment, teleconferencing, medical rehabilitation, sports, and gaming purposes, among others [Slater and Sanchez-Vives, 2016]. Immersive environments can lead to more entertaining and engaging experiences, where VR, compared to AR, provides a more immersive experience, but more discomfort and disturbance at the same time [Carmigniani and Furht, 2011]. Furthermore, it has been shown that these technologies can potentially lead to games that provide more effective learning experiences [Akçayır and Akçayır, 2017], and faster and better medical rehabilitation [Ma et al., 2014].

Along with, and in part due to, these advancements, visual tracking algorithms have also seen countless breakthroughs, mostly thanks to deep learning. It is feasible nowadays to track one's facial landmarks [Chang et al., 2019], body pose [Sun et al., 2019], or hand configuration [Boukhayma et al., 2019] in real-time using modest equipment. Combining immersive and tracking technologies offers exciting new mechanisms to engage with the user. On the one hand, various poses and expressions can be identified, and used as a means of communication for the user, offering more intuitive control and language [Piumsomboon et al., 2013]. On the other hand, accurate and fast positioning can be exploited for visual augmentation and immersive game-play.

As a consequence of these technological advances, it is now possible to consider body position and body movement in avatar design and avatar animation. In this chapter, we explore the symbiosis of hand input, that is avatar movement, and hand display, that is avatar appearance. Specifically, we present ***DigiGlo***, an embodied interaction mechanism emphasizing and utilizing the connection between the user's hand and its digital counterpart.



(a) (b)

Figure 3.2: *Concept art for **DigiGlo** depicting different settings. (a) Virtual reality setting. (b) Spatial augmented reality setting.*

In the explored setting, our most instrumental organ for interaction with the environment [Wilgis, 2014] is used both as input **and** display. Hands have been used to control games since the Microsoft Kinect, and other parts of our body, the face, for example, have been used as displays before [Hieda and Cooperstock, 2015]. However, to our best knowledge, no game nor learning activity has been developed that uses the palm of a hand both to depict the game and to control it.

We argue that the setting explored by ***DigiGlo*** has several advantages, and have designed activities, shown in Figure 3.1, to evaluate them. In addition to the novelty of the proposed gaming mechanism, these activities examine three main advantages:

Embodiment. As described in the previous chapter, learners respond differently and more effectively to movements and bodily interaction, compared to only seeing and listening [Kirsh, 2013]. In this chapter, we exploit these concepts for the design of educational games that portray objects on the player's hands, requiring some physical interaction.

Intuitive Control. As the system parses raw hand motion, one can move the hand naturally. For example, when trying to move an object in the game, the player fully and naturally understands the expected end position of the moved object.

Split Attention. Typically, when using the hands for control, the user's gestures control effects that are seen somewhere else in the virtual world, forcing the user to look at the target, without observing the actual motion performed by the hand. This phenomenon is called *split-attention*, which has been shown in the past to impair concentration [Sweller et al., 2011]. Seeing the game content on the hand, which also controls it, alleviates this problem.

To demonstrate the value and potential of a unified hand input and display system, we have designed three playful activities, described in detail in Section 3.3: *Space Traveller* examines the intuitive control offered by the setup, *Marble Runner* addresses split attention, and *Noelle's Ark* focuses most on the concept of embodiment for learning.

In this work, we focus on a VR implementation of ***DigiGlo***, as depicted in Figure 3.2a, where the user is wearing a VR Head-Mounted Display (HMD) to experience the activities. The activities however are equally appropriate for an AR setting, or more specifically for Spatial Augmented Reality (SAR), such as shown in Figure 3.2b. In SAR, digital content is laid over a real-world environment using projector-based illumination. This scenario has several advantages over traditional VR. These include a more natural, nauseous-free integration of the digital and physical worlds, the lack of equipment worn by the user, and user-friendliness for developmental or clinical populations. Using SAR on humans in entertainment has unquestionable advantages, and has been used in famous shows, such as *Lady Gaga's Tribute Performance* [BizTech, 2016], Disney's *Frozen on Ice* [Times, 2019], and others [Asai, 2019]. Employing SAR for unknown moving surfaces, such as hands, is challenging. As such, a system able to perform this task for novel performances of hands does not exist, as far as the authors are aware of. That said, research in the field has already proven this to be possible for other scenarios, such as facial performances [Bermano et al., 2017], and clothing deformations [Narita et al., 2016]. We therefore believe that a system for projection on hands can indeed be developed. For this reason, we have designed our activities to display only flat content on the palm itself and consider this research to also provide motivation for developing such a system.

Through our three activities, we have evaluated the potential benefits of this interaction mechanism and the potential benefits of ***DigiGlo***. We have conducted a preliminary usability study using one of the activities, followed by another study that included all activities, and in-depth interviews. The latter has been done on a smaller scale due to the current world-wide emergency situation. Through these studies, we observe high acceptance of the mechanism by novice and experienced users alike and drew a set of recommendations that should be considered when designing the next system, or applications for it. Finally, we believe that introducing this concept to the gaming and learning worlds could be quite impactful, and that this work poses as enough motivation for developing the aforementioned SAR system.

3.2 Theoretical Foundation

As described in Chapter 2, the human body plays an important role in how we perceive and communicate with the environment. Indeed, for thousands of years, human beings have been interacting and exploring the world relying mostly on their perceptual and motor systems. Hands, in particular, are a crucial tool for supporting cognition and communication: we use our hands to learn how to count, we support speech with hand gestures.

Focusing on hands specifically, ***DigiGlo*** builds upon several key components: a theoretical foundation upon which we base our conjecture that the proposed system is beneficial, the concept of hands as an input mechanism, and the concept of hands or other body parts as display. In the following, we address the state-of-the-art for each of these components and discuss the implications for our avatar design.

3.2.1 Control and Hand Gestures

Hand-based controls have already been explored for digital activities. In this context, the mode of interaction between humans and computers is mainly dictated by technology. For decades the predominant form is a keyboard and a mouse. However, technological advancements have made the interaction less restrictive, with examples such as the Wiimote controller [Gallo et al., 2008], or the more recent vision-based tracking from a monocular camera [Boukhayma et al., 2019]. These advancements make the interaction through hand gestures more *natural* [Chu and Begole, 2010] and engaging [Li, 2016]. Since the release of the Leap Motion sensor, much work has been focusing on evaluating the merits this technology can have on human-computer interactions. Studies have demonstrated that the interactiveness of hand gestures relies on accuracy [Marin et al., 2016], and has the potential to enhance the learning of sign languages [Potter et al., 2013], to help with rehabilitating stroke patients [Khademi et al., 2014], and even to improve shopping experiences [Chu and Begole, 2010]. Piumsomboon et al. [Piumsomboon et al., 2013], for instance, constructed a set of usable gestures using an elicitation survey [Piumsomboon et al., 2013], and recommend using these for AR design. None of these works, however, have looked into the gestures' usability when a split-attention effect is not present. This is in contrast to our studies, which evaluate similar gestures but eliminate the confounding factor of split-attention.

In the context of gaming, Pirker et al. [Pirker et al., 2017b] provide evidence of higher levels of engagement when using hand gestures, but point out that this mode of interaction induces exhaustion. Khademi et al. [Khademi et al., 2014] demonstrated that turning a physical task into a hand gesture game can motivate stroke patients to faster rehabilitation, and Silva et al. [Silva et al., 2013] have shown that the Leap Motion is a powerful tool to simulate musical instruments [Silva et al., 2013].

In conclusion, although hand gestures can raise motivation and engagement with playful and educational activities, it can also cause physical fatigue. Moreover, hand-based controls heavily depend on the underlying tracking technology. This is particularly important in relation to the design implication related to the sense of embodiment, as described before. During our study, we have witnessed similar reactions in terms of high engagement at the cost of more bodily stress.

3.2.2 Projection Mapping and Body Display

Another major aspect of ***DigiGlo*** is the body display. Generally, digital playful activities are not restricted to screens. Mapping the activity's content onto physical objects makes the activity more tangible and allows us to create novel gaming experiences.

With *Inner Garden*, Roo et al. [Roo et al., 2017] show how projection mapping can turn a simple physical sandbox into a meditative environment to achieve a state of mindfulness and focus on one's body. Following up on this work, the *VRBox* system makes use of VR to provide new interactive capabilities to the sandbox user, and demonstrates how these support playfulness and creativity [Fröhlich et al., 2018]. In both examples, the added value is created by letting users physically interact with the display, i.e. with the sandbox, using their hands. ***DigiGlo*** further improves the interactivity by mapping the display directly on the users' hands.

Mapping digital content to the body is not new. *Body display* is the concept of displaying virtual content on the player's body, for example, using VR or SAR. Bermano et al. [Bermano et al., 2017] use the user's face as both controls and display to achieve real-time digital make-up. The position as well as the facial expression is captured allowing the system to simultaneously adjust the rendered graphics and project them back onto the player's face. The novelty of this work lies in the low latency achieved and the fact that the system does not require facial tracking markers. This enables a range of new possibilities.

Similarly, Hieda et al. [Hieda and Cooperstock, 2015] present *SharedFace*, a system enabling users to digitally draw on their faces using only their hands. This system uses the face as a deformable display and the hands as controls. Even though the system was tested with hundreds of users, this work does not include results regarding users' perception and enjoyment.

Only limited work exploring *Body Display* for gaming and education exists, and to the extent of our knowledge, mapping digital content onto the palm of the hand has not been explored yet.

3.2.3 Embodiment

As described in Chapter 2, we consider three forms of embodiment: embodied cognition, embodied interaction, and avatar embodiment. In this section, we describe the implications of each of these perspectives on avatar design, in the light of the state-of-the-art for hand-based control and body display.

From the perspective of embodied cognition, gestures should be congruent to the concepts that they embody or act upon. Specifically, as we reconnect avatar appearance and avatar movement, we ought to ensure that the meanings highlighted by each aspect are consistent with one another. For example, in our *Noelle's Ark* activity, when a heavier object is represented on the digital hand, the interaction invites the learner to lower the corresponding physical hand.

Regarding embodied interaction, we predict that users will have a more engaging experience when using gestures that are as meaningful as possible physically. This is most evident in the activities *Noelle's Ark*, where the hands imitate the twin-pan-balance device, and with *Marble Runner*, where the physical position of the hand emulates the same position in the virtual world. We also hypothesize that this advantage has a significant impact on usability.

Moreover, we build on the *Leib*, e.g. feeling body, perspective [Mueller et al., 2018] by integrating positive gestures to highlight positive events, for example a thumbs-up gesture to start a new level in *Space Traveller*, and by considering embodied meaning, for example closing the hand to save an object in *Noelle's Ark*.

Finally, reducing spatial disparities between the virtual body and the physical body is an important aspect of the sense of avatar embodiment as it supports the sense of self-location. But this also has importance for cognition. The split-attention effect occurs when one tries to learn while needing to pay attention to two or more sources of information, either spatially or temporally [Mayer and Moreno, 1998]. This effect can impair learning, and hence reconnecting the information sources, both spatially and temporally, is advised [Sweller et al., 2011].

DigiGlo offers exactly this connection. Displaying the content on the controlling hand places the information at the same spatial point, and lets the users see their own movement along with their effects, thus supporting the sense of embodiment while eliminating split-attention effects.

Other aspects of importance in avatar embodiment are the sense of agency and the sense of body ownership. Regarding the sense of agency, we need to align body movement and avatar movement. This aspect is mostly tied to the tracking technology used, also of important for hand-based controls in general. Regarding the sense of body ownership, it is often recommended to align the appearance of the digital avatar with the appearance of the user's body. However, altering the avatar can also result in positive outcomes in terms of behavior and learning [Banakou et al., 2018; Kilteni et al., 2013]. In our context, we exploit this trade-off by using the appearance of the avatar to display the content of the activity.

In conclusion, when designing the activities for ***DigiGlo***, we align the meaning of the avatar appearance with the meaning of its movement, we select physically meaningful gestures accounting for the *Leib* perspective, and we support the sense of embodiment by spatially and temporally aligning the digital avatar with the physical body of the user.

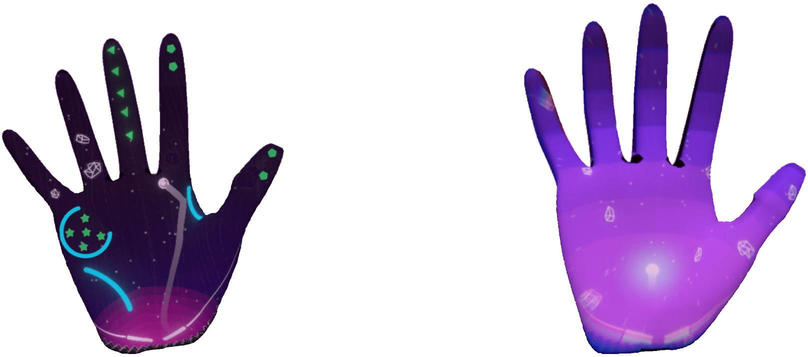
3.3 System and Activities

In this section, we present the design and implementation of ***DigiGlo***, including its three playful activities, *Space Traveller*, *Marble Runner*, and *Noelle's Ark*. These activities are designed to explore different aspects of our system. *Space Traveller* exploits the fact that the display is hand-shaped in a self-contained experience. *Marble Runner* is interested in the hand as a controller to explore the world, resulting in our view-port metaphor. *Noelle's Ark* focuses on embodiment for education, using the whole upper-body as a metaphor for a twin-plate balance. For each activity, we chose gestures following guidelines from the literature and exploiting our hand input-display mechanism when possible. The gestures were then refined through iterative processes with users.

3.3.1 System

We employ an off-the-shelf hand tracker (Leap Motion sensor) for game control and a VR HMD (HTC Vive) for display. The Leap Motion sensor is attached to the VR headset, a configuration that comes pre-calibrated with the hardware. We implemented all the activities using the Unity game engine.

As mentioned in Section 3.1, we have designed ***DigiGlo*** to be compatible with a potential SAR setup as well, in order to assess its potential utility. For this reason, our content is presented only on the surface of the virtual palm, and the chosen color schemes are of high contrast. In the following, we present the different activities implemented for ***DigiGlo***.



(a) **(b)**

Figure 3.3: Space Traveller *game visuals. (a) Sample level showing the player's ship with a trail, green fuel containers, blue bumpers, and wire-framed asteroids. (b) Hyperspace jump, the transition between two levels.*

3.3.2 Space Traveller

Space Traveller is a pinball game on a hand-shaped playfield. The purpose of this implementation is to tie custom control gestures to the game setting. Through these controls, we demonstrate how using the palm as the display can naturally give intuition to complex movements, and how to avoid triggering a spatial split-attention effect.

Narrative

The player steers a spaceship to explore the broad expanses of the universe. The journey is dangerous and requires a lot of fuel. Therefore, the player must collect all fuel containers while avoiding asteroids and wormholes to trigger the next hyperspace jump, which starts the next game level.

Goal

This game uses a score-based objective. Collecting one fuel container awards one point while hitting an asteroid removes one point. The game ends when five ships are lost, or when the player completes the eighth, final level.

The levels are designed such that all the fuels can be collected in one hit of the flippers in order to inspire the players to think the levels through and grow a sense of mastery. The levels become increasingly difficult, starting by demonstrating each game mechanic and then combining them.

Figure 3.3a illustrates a sample level that contains all game elements: bumpers (blue arcs), fuel (green), spaceship (white with trail), asteroids (white chunks), flippers (white bars), and wormhole (black, emitting purple glow). Figure 3.3b depicts the hyperspace travel animation, displayed between levels. The game also includes sound feedback for the player's actions, such as collecting fuel or crashing into an asteroid.

Controls

Figure 3.4 shows the gestures to control the game. *Thumbs-Up* starts a new round of the game, *Fingers Curl* moves the flippers, *Pinch* is used both to transport the ship from the thumb to the index finger and to spawn a new ship, and *Hand Rotation* influences the direction of gravity.

Novelty

The game's controls are specifically designed for the hand-shaped playing field and give the sensation that the hand is the pinball machine itself. For example, the non-static playing field allows the player to connect the index

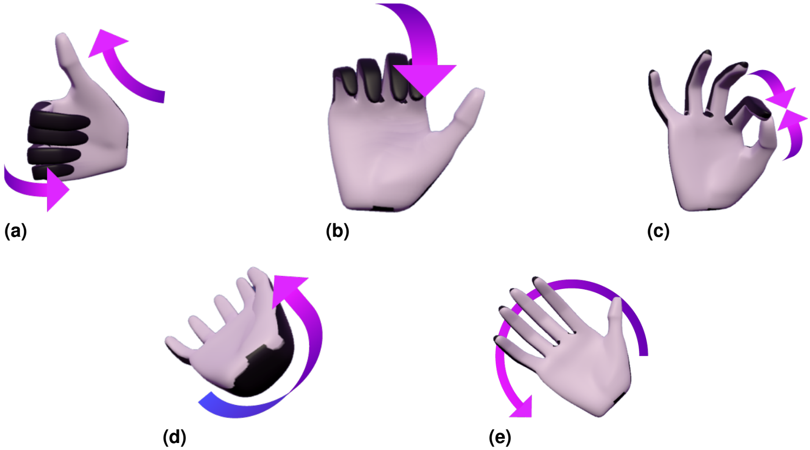


Figure 3.4: *Hand gestures for* Space Traveller. *(a) Thumbs-Up starts the level. (b) Fingers Curl triggers the flippers. (c) Pinch is used both to transport the ship from the thumb to the index finger and to spawn a new ship. (d), (e) Hand rotation affects the direction of gravity.*

finger and the thumb to create a passage through which the spaceship can fly, thus offering a super realistic experience [Wigdor and Wixon, 2011]. We will further refer to this gesture as the Passage gesture. This new mode of interaction does not require the player to actively learn a new gesture and its function since it is intuitive that the spaceship can fly through connected areas. Thus, the player only needs to be aware that this interaction is possible. Certain levels require the player to perform this gesture to collect fuel containers that are located on the thumb.

Finally, *Space Traveller* also offers an interactive version of the traditional gravity mechanic: The wormhole located at the wrist of the hand attracts the spaceship and pulls it downwards. Additionally, the players can move the direction of gravity by tilting their hand. With this interactive mechanism, the player is able to steer the spaceship towards specific objects and to dodge other objects. On some levels, the fuel collectibles are only reachable through the use of this concept. This is an example of a mechanic that would be less intuitive and more cumbersome if the hand and the display were decoupled, as it would create a strong spatial split-attention effect.

3.3.3 Marble Runner

Marble Runner is a rogue-like game where the movement of the main character, the marble, is controlled by the player's hand translation. The marble is fixed to the base of the hand, and hence moving the hand translates to moving the marble in the virtual world, exactly like one would move an object in the physical world.

Narrative

Marbles are meant to roll! On this spiky planet, however, it proves to be a challenge. Using all your agility, stay on the safe path, and shoot at the pointy enemies to roll to the next level! How far will you go?

Goal

The goal of the marble is to survive for as long as possible. Figure 3.5 presents the two phases composing each level. The first phase contains a labyrinth surrounded by spikes that force the marble to stay on the path or lose life energy. During the second phase, the marble faces enemies and traps in an arena. By destroying enemies, the marble gains life energy. In each level, the labyrinth path moves faster and the enemies and obstacles in the arena fire at a higher rate and with greater force than in the previous levels. The game ends when the marble's life energy is lost.

Controls

The player starts the game by curling the index finger. By moving the hand left and right, the player moves the marble left and right while the labyrinth path passes below the marble at a speed that increases in each level. The player can give small acceleration boosts to the marble or slow it down momentarily by moving the hand forward or backward. Finally, the player can fire a shooting star to destroy enemies by curling their thumb during the area phase. The initial gesture to fire bullets involved curling the index, mimicking pulling a trigger. However, this gesture was reducing the field of view in a meaningful area of the game display, so we replaced it by the thumb curl gesture.

(a) (b)

Figure 3.5: Marble Runner *game visuals. (a) The marble follows a treacherous path in the labyrinth. (b) The marble is pitted against charging enemies and obstacles in the arena.*

Novelty

This game builds on two novel mechanics. First, the marble is directly controlled by the position of the player's hand. Second, the hand is used as a viewport into the game world. The player only sees the area around the marble and discovers the rest of the labyrinth only by moving the hand. As it feels natural to move an object placed in your own hand, *Marble Runner* offers intuitive control to players by placing the game's central object virtually into their hand and letting them control it directly through their hand.

3.3.4 Noelle's Ark

Noelle's Ark is an educational playful activity to help children learn about the weights of different objects. This activity builds on the principles of embodiment and benefits from intuitive hand gestures as well as reduced split-attention.

Figure 3.6: Noelle's Ark *game visuals. The player needs to show which object, a bike or a Goldfinch, is heavier by mimicking a twin-pan balance with the hands.*

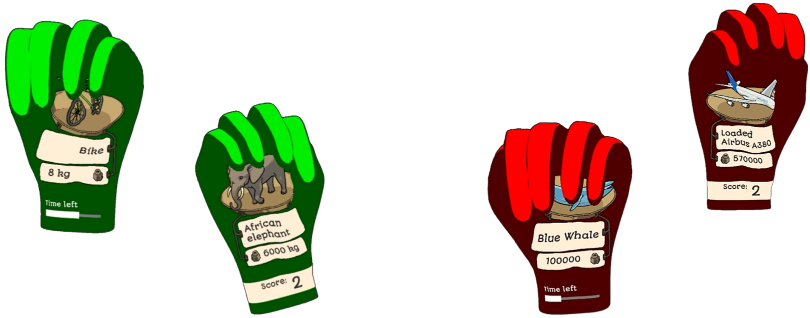
Narrative

Planet Earth is losing its last fight against global warming. Noelle built a space-ark to save as many objects and animals as possible. Unfortunately, the ark cannot carry too much weight, therefore the player must help Noelle figure out which objects are lightest.

Goal

The goal of the user is to correctly evaluate as many pairs of objects as possible in 1 minute. For each pair, one object is displayed on the left hand and another object on the right hand, as depicted in Figure 3.6. The user then mimics a twin-pan balance and moves the hands to reflect the weight relationship between the two objects: the hand containing the heavier object should be lower than the hand containing the lighter object. If the user guesses correctly a score point is awarded and the hands turn green, shown in Figure 3.7a. If the user guesses wrong, a point is lost and the hands turn red, as depicted in Figure 3.7b. By displaying the correct weights of the objects after the user guesses, the game gives the user the opportunity to learn the objects' weights.

The game includes objects weighing from 4 grams (a pea) to 5.75 million tons (the Great Pyramid of Giza). The comparisons are generated randomly from a pool of objects, but the maximum weight difference between the two compared objects decreases with time, making the activity increasingly difficult.



(a) **(b)**

Figure 3.7: Noelle's Ark *game visuals. The virtual hands turn green or red to indicate that the player's guess was correct (a) or wrong (b), respectively.*

Controls

The game is launched using the Thumbs-Up gesture. The scale pans are displayed on the hands of the user and are moved accordingly. Finally, the user can finalize an answer by closing the fingers of both hands as shown in Figure 3.7.

Novelty

The role of this activity is to offer a playful way to learn about weight comparisons. The design of this activity is inspired by three main aspects described in the literature. First, the work on the *Mathematics Imagery Trainer* [Howison et al., 2011] notes how moving one's hands meaningfully can help to understand the concept of proportions. We follow the same approach for weights. Second, because the hand movements have a direct physical meaning, which is weight and gravity, our activity follows the recommendations of using hand gestures with a physical meaning over other kinds of gestures [Piumsomboon et al., 2013]. Finally, because the pans of the balance are displayed directly on the hands that are used to move them we avoid the spatial split-attention effect that might occur with a screen-based desktop setup [Sweller et al., 2011]. The combination of these three aspects is enabled by our novel palm input and display mechanism.

3.4 User Studies

We have conducted two user studies to identify the benefits of ***DigiGlo*** and guide the implementation of future systems.

The first study focuses on evaluating the overall usability of ***DigiGlo***, while the second study provides an in-depth analysis of the system and extracts design recommendations for future implementations.

3.4.1 Preliminary Usability Study

In our first study, we focused on evaluating ***DigiGlo***'s usability, using our activity *Space Traveller*. This activity best reflects the novelty of our palm input and display mechanisms. The field is hand-shaped, and various kinds of novel hand inputs are directly linked to the coupling between palm input and palm display, such as the gravity, controlled by the hand orientation, and the Passage gesture, virtually connecting areas according to those physically touching.

Protocol

The study took place in our lab. Each participant was presented with an instruction sheet explaining the narrative and the goal of the game as well as the different gestures involved. The participants were allowed to ask questions if anything was unclear. The participants played the game for 10 minutes. After the play phase, the participants filled in several questionnaires: the System Usability Scale questionnaire [Brooke and others, 1996], a Game Experience Questionnaire [IJsselsteijn et al., 2013], an immersion questionnaire [Jennett et al., 2008], and gestures-specific items and demographic questions. The gesture items were based on a 5-points Likert scale and followed the template of these examples: “The Passage gesture was easy to use” and “I enjoyed using the Passage gesture”. The possible answers ranged from “strongly disagree” (1) to “strongly agree” (5).

Demographics

24 people participated in this study. 7 participants identified as female, and 17 participants identified as male. The average age was 31.6 ($s = 14.9$, range 13 to 69 years). 12 participants indicated having high experience with computer games, 5 indicated having moderate experience with computer games, and 7 participants indicated having no previous experience. 22 participants were right-handed and 2 were left-handed.

Results

Our system ranked 77 on the System Usability Scale (SUS) ($s = 12$, range 53 to 100), which can be qualified as "Good" [Bangor et al., 2008].

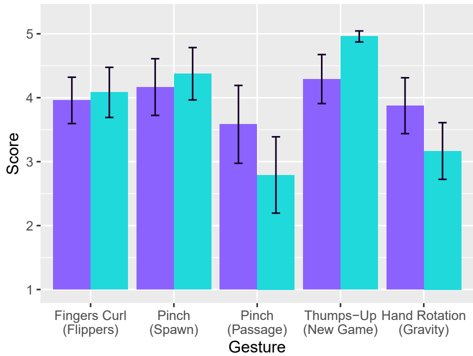


Figure 3.8: *Evaluation of the different gestures for item "I enjoyed using this gesture" (purple) and "I found this gesture easy to use" (cyan). Range: 1 - Strongly disagree, 5 - Strongly agree. The black bars represent the 95% confidence interval.*

We analyzed how participants ranked the different gestures. Figure 3.8 summarizes the scores. For the *enjoyment* criterion, the Mauchly's test for sphericity was not significant ($W = 0.55$, $p = 0.18$), so we performed a within-participants ANOVA ($F(4, 92) = 1.72$, $p = 0.15$, $\eta^2 = 0.051$) and found no significant effect of the gesture on the enjoyment score. Regarding the *ease of use* criterion, sphericity was violated ($W = 0.35$, $p = 0.008$, $\epsilon = 0.71$), so we report the Greenhouse-Geisser corrected results: $F(2.83, 64.99) = 20.41$, $p = 3e{-}9$, $\eta^2 = 0.42$). The ANOVA reveals significant differences between gestures, so we performed post-hoc pairwise

Bonferroni-adjusted t-tests. We also compared different kinds of gestures using an ANOVA with orthogonal contrasts. These tests revealed that non-physical gestures were found easier to use than physical gestures, contradicting our initial expectations and previous research in the field [Piumsomboon et al., 2013]. Specifically, the Passage and the Rotation gestures were found less easy to use compared to others ($p < 0.01$ for all comparisons). The difficulty with the Passage gesture comes from the fact that it is used in a challenging moment of the game, where the player first needs to aim at the right finger, and then perform the gesture with perfect timing. Additionally, the tracking was unfortunately not always accurate and caused some frustrations. Indeed, when the same gesture was used for spawning, it was significantly better appreciated as it happened at a less intense moment. The issue with the Rotation Gesture is identified in the second study. However, even if these gestures ranked lower, they were still appreciated and received positive qualitative feedback: "The Passage gesture is really cool, it would be awesome if it worked with all fingers", "The Passage Gesture was pretty hard to use but it's a cool feature". Furthermore, symbolic gestures were found easier to use than abstract ones. This might be due to the fact that the Thumbs Up is a well-known gesture, is performed at a calm moment of the game, and has a positive connotation when we consider a *Leib* perspective [Mueller et al., 2018]. In addition, we performed a covariance analysis of the game performance and gesture scores. The ANCOVA revealed no significant effect of the player's performance on the gestures scores ($F(1, 114) = 0.064$, $p = 0.80$, $r = 0.024$).

We now present the results of the *Game Experience Questionnaire*, aggregated against the different components. The results range from 0 - "Not at all" to 4 - "Extremely". Due to an experimental error, the *Positive Affect* component is computed over one item only. Participants reported in average a score of 2.21 ($s = 0.76$) on the *Competence* component, 2.75 ($s = 0.79$) on the *Sensory and Imaginative Immersion* component, 3.29 ($s = 0.80$) on the *Flow* component, 2.58 ($s = 0.79$) on the *Challenge* component, 0.64 ($s = 0.58$) on the *Negative Affect* component, 3.33 ($s = 0.70$) on the *Positive Affect* component. We compared the results for different gender groups, age groups, and gaming experiences groups using ANOVAs and post-hoc t-tests. We found a significant difference between the group with less gaming experience and the group with more gaming experience on the *Sensory and Imaginative Immersion* item. Participants with higher gaming experience felt less immersed than those with lower gaming experience ($p < 0.05$). We believe participants with gaming experience had higher expectations as they are more used to virtual environments.

Regarding the *Immersion Questionnaire*, possible answers ranged from 1 -

"Not at all" to 5 - "Very much so". Participants reported in average a score of 4.08 ($s = 0.39$). We compared the results for different gender groups, age groups, and gaming experiences groups using independent t-tests. These tests revealed a significant difference between participants younger than 40 years old, and older ($p = 0.048$). Older participants felt more immersed ($m = 4.30$) than younger participants ($m = 4.03$). This might be due to the fact that older participants have less experience with video games and virtual reality systems.

3.4.2 System Design Study

The goal of the second user study is to evaluate the potential of ***DigiGlo*** through different activities and to gather feedback for future iterations of the design process, following a research through design approach [Zimmerman et al., 2007].

Protocol

We conducted the study with each participant individually. Each participant started by answering questions about their profile and their experience with technology. Afterwards, the study was split into two parts of 30 minutes each: *test* and *interview*.

During the *test* phase, the participants played the three activities in random order. Before starting an activity, the participants read a document describing the activity and the different gestures involved. The participants were given the opportunity to play each activity several times for a total of 8 minutes per activity. During the first attempt, the experimenter provided guidance to the participants with respect to the interaction techniques and the goal of the activity. This ensured that all the participants comprehend the activity. The participants were asked to provide think-aloud comments as they went through testing. The experimenter took notes about the participant's comments and behaviour in order to gather in-game data about the participant's experience, as it might be difficult for them to recall all the details afterwards.

During the *interview* phase, the experimenter started by asking the participant various questions, about the hand gestures, the hand display, ***DigiGlo***, the game experience, and further uses of the system. This was followed by an open conversation with the participant. During the whole session, pictures of the different activities were available to the participant, to help them recall the experience. The entire interview process was recorded.

Demographics

Table 3.1 summarizes the profiles of the participants. All participants are right-handed. P4 used a Valve Index VR HMD instead of the HTC Vive. P2 has limited vision in one eye and reduced perception of 3D effects but insisted that this did not affect his experience negatively. P3 also participated in our preliminary study and hence had a deeper understanding of our system and experienced reduced novelty effect. P2 and P3 mentioned hobbies requiring high body awareness and provided useful feedback regarding the ergonomic aspects of ***DigiGlo***. P1 and P2 had low experience with video games and VR systems while P4 and P5 used them very often.

Method

We performed our analysis using an inductive thematic approach [Braun and Clarke, 2006]. First, we transcribed the interviews and think-aloud comments. We coded the different items of the dataset with the topic they addressed, for example, "Natural hand gestures" or "Physical discomfort". Through iteration, we identified several main themes, for example, "Hand input" or "Sense of body". We qualified themes as most relevant if they were mentioned by several participants, if they were mentioned in think-aloud comments without explicit questioning from the interviewer, and if they were related to the expertise of the participant (e.g. body awareness and artistic sensibility).

3.4.3 Discussion

We analysed the results of the observation and interview process and identified several main items of consideration for ***DigiGlo***. Below we present a summary of our findings and offer design recommendations to address each aspect. To illustrate each point, we quote participants comments, translated to English or edited for readability when necessary. We then list different suggestions for promising ***DigiGlo*** applications.

Select Meaningful and Simple Hand Gestures. Overall, participants enjoyed the hand gestures and movements. They found them very natural: "It's the easiest, it's natural. You don't have to think that much. It's simple to interact", "My hand is free", "It's gadget-less. It's as simple as possible to interact" (P5), "I liked it. I really enjoyed it in fact" (P1). Some participants even found these activities to be a great way to exercise the dexterity of their hands: "My favourite game is *Space Traveller* because it's the one that required the most dexterity with my hand" (P3).

The participants enjoyed simple, natural gestures more: "The simpler, more intuitive gestures are closing the hands, or do a Thumbs-Up" (P4), and struggled with more unusual gestures: "but curling the index with an open hand is not very natural" (P4). Participants expected consistent gestures across the different activities: "I think a standardised way of starting the game makes sense" (P3). Participants also enjoyed gestures with a meaningful explanation. For example, in *Noelle's Ark*, the movement of the hands according to a twin-pan scale was mostly appreciated: "It feels more natural and feels like a scale" (P2), "this control gives you the explicit explanation that heavier things are more difficult to carry therefore they go down" (P3), even though they would have preferred to have their hands horizontal to push the metaphor even further.

During the *test* phase, all participants had the chance to try each activity once in order to familiarize themselves with the gestures, and we noticed that in most cases, they did not need more trials to achieve this goal. We also noticed that during the interviews, the participants used the gestures of the games as part of their description of the game. This shows the potential of ***DigiGlo*** from an embodiment standpoint, for example, for mathematics education where the use of gestures can improve students' understanding [Nathan and Walkington, 2017]. In particular, (P3) mentioned that ***DigiGlo*** helped him "feel the problem-solving".

We conclude that ***DigiGlo*** activities should exploit a vocabulary of gestures that are simple, meaningful, and consistent across activities. For educational activities, in particular, the gestures should be strongly connected to their effect. These findings are consistent with previous hand gestures design guidelines [Piumsomboon et al., 2013]. However, with ***DigiGlo***, new gestures can be qualified as meaningful. For example, a rather abstract gesture like pinching the index and the thumb to bend the playfield becomes meaningful when the playfield is displayed on these fingers. In our work, we present a first exploration of hand gestures within the ***DigiGlo*** paradigm, but we believe future work should explore a wider range of gestures in order to generate a language of meaningful gestures in this context. Considering gestures involving both hands is particularly interesting in that context, as connecting the hands also accounts for a connection of the game space. Exploring asymmetrical configurations could also be interesting, for example using one hand as a tool to interact on the other one.

Table 3.1: *System Design - User Study: Participants profiles.*

ID	Age	Gender	Profession	Hobbies	Video games	VR
P1	30	F	Software Engineer	Gardening, Reading, Exercise	Low	Low
P2	30	M	Software Engineer	Skiing, Hiking, Reading	Low	Low
P3	32	M	Security Engineer	Music, Brazilian Jiu-Jitsu, Yoga	Low	High
P4	28	M	Technical Artist	Video games, Music, Drawing	High	High
P5	30	M	Software Engineer	Reading, Developing apps, Learning German	High	High

Train and Calibrate Hand Gestures. In our study, the participants read a descriptive document before playing an activity. The document contains a list of gestures as well as their effect in the game. Many participants did not like this way of learning gestures, and struggled to perform them properly on their first attempts: "I would have liked more progressive instructions as well as a tutorial instead of a document" (P4), "The challenging part was remembering the movements that you need" (P1).

Moreover, the neutral hand position for P2 and P3 is with curled fingers. This triggered undesired events, like the flippers in *Space Traveller* and the comparison validation in *Noelle's Ark*: "I didn't feel the need to stretch my fingers, which is less natural for me" (P2), "I think for me the challenge was to keep my hand actively flat" (P3).

To solve these issues, it is important to include a tutorial phase at the beginning of the activities to let the users get used to the gestures, or to calibrate the gesture recognition.

Adapt to the Hand Display. The hand is a small surface to display content on. When asked to imagine applications for ***DigiGlo***, many participants found this to be a restricting factor and wished to expand the display surface: "I would expect to use that not only to project on my hand but also on the environment" (P5), "I would like this on my whole body" (P1), "You should also have the back of the hand" (P4). However, with respect to the activities, they only found the display restrictive when playing *Marble Runner*: "Hard to tell when the path will stop" (P2), "If I want to move back I don't see what's behind me so I'm just moving blindly" (P3), "It felt like I had more load on my mind because I had to move the screen. There's no kind of reference point, it's just an abstract screen" (P1). To summarize: "In *Marble Runner*, the surface is mainly small. *Space Traveller* was fun because it's contained, and everything is taken into account." (P1). *Marble Runner* is a fast-paced game, and the hand is used as a viewport on an ever-changing terrain. The combination of high-frequency visual updates and fast-paced gestures overwhelmed the users. We believe that the viewport metaphor could still be interesting to explore, with two modifications: the activity should be slower-paced (for example, by zooming out more), and the main point of focus should be located at the center of the hand, rather than at its root.

Several participants mentioned an interest in using both hands, either as a duo palm input-display mechanism or as a mean to decouple input and display: "I would like to have both hands, to do something with both" (P1), "I think that using one hand for input and one hand for display would greatly simplify understanding" (P4). In this chapter, we are focusing on the palm input-display mechanism, so we will only analyse the first option. *Noelle's Ark* implements this approach, with simple gameplay and a limited set of gestures. Yet, some participants struggled to grasp all the information: "You can't look at your left-hand wrist: you lose focus on the object and it's mentally demanding to focus on that" (P3). In general, we noticed that the wrist area is difficult to focus on as some participants even asked the interviewer to read it for them, but this issue was amplified in the two-hands set-up. We would advise against displaying crucial information in this area.

Only P4 mentioned that he would have enjoyed 3D objects for *Noelle's Ark*. For the other participants the quality of the visuals seemed less crucial for the experience. However, as P4 is a professional 3D technical artist, we believe that his perspective is interesting to explore in future work.

In order to account for the limited display space, it is important to carefully design the activity within and with respect to the hand shape. Using both hands can solve part of this issue, but implies limited focus available for each hand. Future work could also explore magnifying the hand in VR, with special care for possible negative effects on the sense of embodiment.

Combine Hand Gestures and Display. ***DigiGlo*** uses the hand both as a display and an input. This enables some novel game mechanisms, based on intuitive and natural controls: "The User Experience is very nice, it's so natural, you don't feel the game imposing so much" (P5), "I have to solve a problem and then my hand does whatever it needs to do to solve the problem, you have this high-level idea 'I need to go right' and it just happens" (P3), "I wonder why there are no other games like this" (P1). In this study, unlike in the preliminary study, the participants enjoyed the Passage Gesture in *Space Traveller* as it made the best use of the palm input-display mechanism: "I like it, I feel like I am curving space and time" (P3). We believe that these controls are particularly appreciated because of the reduced spatial split-attention effect.

However, ***DigiGlo*** also raises specific design constraints. Participants suggested that the Fingers Curl gesture used to activate the flippers in *Space Traveller* restricted their experience. Indeed, as they tried to aim for a specific finger, they had to close this very finger: "When you close the hand to shoot, you have to imagine where the spaceship goes" (P4), "You have to fold your fingers and you lose visibility of the game space. The game design needs to make sure that there's nothing relevant that you lose by closing your fist" (P3).

We also noticed that, because the system is based on the hand, the participants had strong expectations with respect to the physics of the games. For example, most of them struggled to grasp how gravity varied as they rotated their hand in *Space Traveller*: "I wish gravity would listen to me more" (P1), "It was about understanding how the physics worked" (P4). When we designed the balance metaphor for *Noelle's Ark*, we realised that keeping the hands horizontal reduced the readability of the content. Instead, we designed the activity so that the hands are positioned vertically, closer to the user's face. However, when playing *Noelle's Ark*, several participants kept holding their hands horizontally (P2 and P3).

In order to make the full use of ***DigiGlo***, the input and the display should be designed in an intertwined manner: one should not limit the other. Moreover, because the hand is used intensively, the users expect the game to follow the laws of physics of the real world and can struggle to adapt to dissonances. This is a form of *Tactile Illusion* [Hayward, 2015]. These illusions arise when touch perception and visual perception are not in agreement. ***DigiGlo*** is particularly prone to this kind of illusions as the visual cues and the hand, our main touch agent, are co-located. More generally, because people are used to using their hands in the physical world, they have strong expectations of how their hands are supposed to behave. Although these expectations can be challenged, for example with the "six fingers hand" illusion [Hoyet et al., 2016], future work should explore how the participants' expectations and preferences for different body configurations influence their experience, and how to design accordingly.

Another important aspect is the effect on the sense of embodiment [Kilteni et al., 2012]. In its current version, ***DigiGlo*** supports the sense of self-location by mapping the game content onto the user's hand and the sense of agency by updating the game world in 100 ms. The sense of body-ownership could be improved by adapting the shape of the virtual hand to the user's hand. Further work should empirically evaluate the influence of these factors on the sense of embodiment.

Provide Multimodal Feedback. Feedback played an important role in the participants' enjoyment and empowerment. The sound feedback in *Space Traveller* helped the participants' awareness: "I loved *Space Traveller* because of the music and how it reinforces the game, for example when I crash into asteroids or go into the wormhole" (P3). The coupling of feedback with hand movement and position was also helpful: "In *Noelle's Ark* I feel like they're re-enforcing the decision: I just have this mapping between green and the position of my hands when I see these two pictures" (P3).

Because the display is limited, it is particularly important to rely on multimodal feedback, including sounds, visuals, and proprioception factors such as body position and body movement. Previous work on multimodal feedback [Cockburn and Brewster, 2005] demonstrates how adequate sound and tactile feedback can improve performance on visual tasks. We suggest grounding ***DigiGlo*** activity design according to this line of research.

Reduce Physical Constraints. The use of ***DigiGlo*** can create physical discomfort if the hand stays immobile or performs uncomfortable gestures, or if the user stares down for too long, especially for participants with high body-awareness: "It puts pressure on the shoulders" (P2), "Our palm has to be constantly facing up, this puts some constraints on your neck" (P3), "As long as I can't keep my head straight and my spine straight it's a problem" (P3), "The hand is always in front for you, it's tiring" (P4). We suggest to design interaction that keeps the hand active, and allows the user to drop the hand or move it to a more comfortable place when necessary: "Having things like *Noelle's Ark* where you keep your hands in front without having to look down is a lot helpful for me" (P3).

Previous work showed that users experience discomfort after playing a hand gestures-based video game for an average of 23 minutes [Pirker et al., 2017b]. As the body of the user is heavily involved when using ***DigiGlo***, we advise being mindful of possible discomforts and design accordingly. For example, future work could explore how to integrate specific movements and pauses in the activity to avoid negative effects on the body. Similarly, exploring different body configurations such as laying down on one's belly is relevant. The body-awareness raised from such experiences can also be positive. In particular, as ***DigiGlo*** enables non-invasive visual feedback, provides space to focus on one's hand, and exists as an extension of one's body, we believe ***DigiGlo*** is a promising system to offer somaesthetic appreciative experiences [Höök et al., 2016].

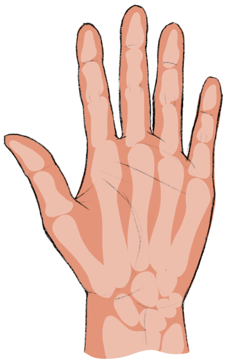
We conclude this section by listing several applications envisioned by the study participants: small multiplayer games with friends (P5), ninja game with hand gestures for incantations (P4), whack-a-mole (P4), nail polish preview (P1), cooking handbook and tutorial (P2), gamified recovery exercises for hand injuries (P5), gamified hand-yoga (P3), fine-tuned gestures trainer for music (P3). More generally, we believe ***DigiGlo*** offers a framework and inspiration for a plethora of future games and activities (Figure 3.9). A few examples are a guitar-hero-like rhythm game in which one plays a virtual musical instrument on the hand using the fingers, a puppeteering game in which the user can make a character dance on the palm, an educational activity for children, teaching them mathematics with their hands [Li et al., 2019], and a hand rehabilitation therapy game. All these could be promising ideas for fun and exciting future ***DigiGlo*** activities.

3.5 Conclusion

In this chapter, we explored how the digital avatar can be designed to support embodied interaction. Specifically, we focus on a novel embodied interaction approach and we demonstrate how the palm of the hand can be used as both an input and a display mechanism for gaming and educational activities. We present Digital Gloves (***DigiGlo***), a system comprising a hand tracker and a Virtual Reality (VR) Head-Mounted Display (HMD), which virtually projects content on the user's hand and lets the user control an activity solely by hand movement and gestures. Through three activities, we demonstrate that ***DigiGlo*** is stimulating and introduces a mode of interaction that warrants further investigation.

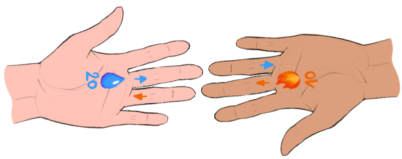
A preliminary user study conducted with *Space Traveller* focuses on the usability of ***DigiGlo*** and demonstrates that participants enjoyed the game and

(a) Music at hand

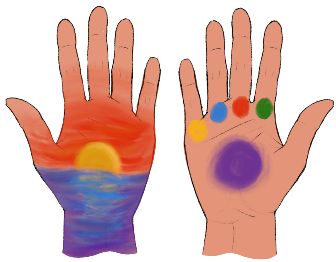


(b) Anatomy learning

(c) Nail polish simulator



(d) Hand-shake transaction



(e) Bi-manual painting

Figure 3.9: *Examples of future activities using the **DigiGlo** mechanism: (a) A music game where the player needs to play the song by folding the respective finger when the note reaches it, (b) An educational activity to learn about the anatomy of the hand, (c) A nail polish simulator, (d) A multiplayer activity where resources exchanged are validated with a hand-shake, (e) A painting activity where one hand is used as a canvas while the other one is used as both a palette and a paint brush.*

its mechanics. A total of 24 participants rated the system with an average System Usability Scale (SUS) of 77 and agree that the majority of gestures were both enjoyable and easy to use.

We conducted a second user study on a smaller scale than originally planned because of the global pandemic at the time of the project. Five selected participants thoroughly evaluated all three activities and participated in a discussion about ***DigiGlo***, its activities, and possible future uses. From the discussions we conclude that ***DigiGlo*** requires activities to involve meaningful and simple hand gestures, which are consistent across all activities. Gestures should be taught and practiced in an in-game tutorial, in order for the users to adapt to the gestures and calibrate their hand movement to the hand tracker's gesture recognition. A combination of visual high-frequency details and fast-paced gesture input can quickly overwhelm users and should be avoided. To account for the limited display space of a palm, activities must be carefully designed to make the best use of the shape of the palm and should additionally include multi-modal feedback, such as sounds and proprioception effects.

Limitations. As prototype implementations, the three activities lack ingame tutorials that help the users become comfortable with the gestures before starting the actual gameplay. *Space Traveller* and *Marble Runner* are implemented mainly for right-handed users and it would be relevant to study how to adjust such activities for left-handed users. *Marble Runner* and *Noelle's Ark* lack sound effects and music. As identified during the user studies, sound and other multi-modal feedback greatly improves the experience and augments the limited display space of the hand. ***DigiGlo*** uses a compelling 3D model of a hand, which, however, never matches the user's real hand accurately. This may lead to a reduced sense of body-ownership. Further limitations in our current implementation motivate a number of future research directions. For instance, haptic feedback integrated into ***DigiGlo*** would greatly improve the sense of immersion [Georgiou et al., 2018] and enable new types of games and activities. Haptic feedback especially enhances embodied learning experiences. Regarding our empirical results, a complementary comparative study to evaluate the effects of embodiment and reduced split-attention on the users would be beneficial.

Outlook. Our VR implementation of ***DigiGlo*** presents the game content to the user on a virtual hand through an HMD. In addition to being an interesting testbed for future embodiment research, ***DigiGlo*** also opens new avenues for research. For example, we believe a Spatial Augmented Reality (SAR) implementation of ***DigiGlo***, where the game content is directly projected onto the users' physical hands, presents a compelling future research direction. A SAR implementation does not require the user to provide any equipment. Instead, engaging with an activity is instantaneous, which makes it well-suited for public spaces, museums, schools, training, and healthcare settings. Hygiene concerns that arise when using VR in public spaces also become obsolete.

More generally, embodied interaction mechanisms such as ***DigiGlo***, focusing on 2-dimensional display, or hand interfaces, focusing on 3-dimensional display [Pei et al., 2022], can be explored for playful activities (e.g. Figure 3.10). Several approaches should be considered such as direct embodiment (Figure 3.10, *Dragon Bites*), indirect embodiment (Figure 3.10, *Puppet Dance*), and gestures based on embodied metaphors (Figure 3.10, *Ocean Clean-up*).

Applications for mathematics sense-making. In this chapter, we followed a general approach, as ***DigiGlo*** has wide implications in the field of embodied interaction design, beyond mathematics education. However, this work also transfers to the specific context tied to our research question:

RQ How to design embodied interaction to support embodied sense-making of mathematics?

Our results show that avatar appearance can be used to support interaction meaning or create novel meanings for specific interaction techniques and gestures. In our framework, this is highlighted by the following path: `Avatar` $\rightarrow$ `Interaction` $\Rightarrow$ `Meaning making` (Figure 2.10). However, in the context of mathematics learning, ***DigiGlo*** can be used to support what we call "semantic avatars". Semantic avatars are digital avatars designed to highlight a specific mathematical meaning, explored through bodily, or, specifically, hand actions (Figure 3.11). In our framework, semantic avatar support the following path: `Avatar` $\rightarrow$ `Bodily action` $\Rightarrow$ `Learning`.

For example, by highlighting fingers in a counting task, ***DigiGlo*** can be used to support meaning-making of arithmetic through object collection [Lakoff and Nuñez, 2000]. Going even further, the avatar can be designed to support counting in cyclic groups by keeping track of previous actions, and even to support other bases by hiding specific fingers. But ***DigiGlo*** has some applications beyond finger counting. For example, following the design of *The Hidden Village*, a game directing students towards using gestures tied to specific geometrical concepts [Nathan and Walkington, 2017], our system could

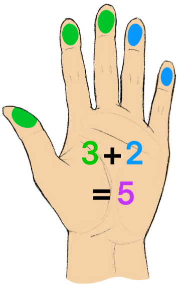
(a) Dragon Bites **(b)** Puppet Dance **(c)** Ocean Clean-up

Figure 3.10: *Three embodied playful activities in Augmented Reality (AR). In Dragon Bites (a), each hand is a dragon, with different food taste. By opening and closing the hands, the player can make the dragon eat food. In Puppet Dance (b), each hand controls one side of the puppet, and can be used to give it certain poses. The player follows a certain choreography in time with the music. In Ocean Clean-up (c), the player tricks a fisherman into cleaning the ocean. By mimicking scissors, the player can cut the fishing hook off. By mimicking a camera, the player can take a picture of a fish: the fish will then pose for the picture and not move for a few seconds. If the hook reaches the sea-floor without catching a fish, it will then grab trash instead. The three activities were designed and implemented by Martina Kessler under my supervision. These pictures illustrate the game concepts and are not in-game screenshots.*

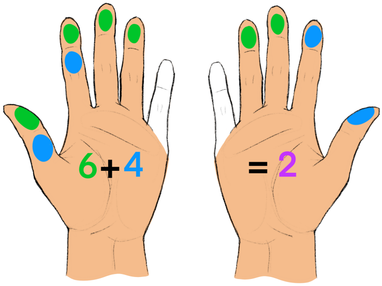
highlight the geometrical meaning on the gestures directly. For example, the hand of the learner could become a tool to highlight specific angular values or general angular behavior by displaying the nature of the angle between the thumb and the index finger. Our mechanism could also be used to implement the *Mathematical Imagery Trainer* [Howison et al., 2011], a learning activity designed to learn proportions by moving the hands in a way that preserves the ratio between the distance of each hand to a reference point. With our system, the reference point could be embodied by the learner directly, and the feedback could be displayed on the hands, rather than on an external screen. With this approach, the concept of proportion can also be apprehended in terms of hand size, rather than only hand position.

Generally, we believe that embodiment research would benefit from considering digital avatars as a mean to convey meaning, rather than only as a manifestation of the user in the virtual world.

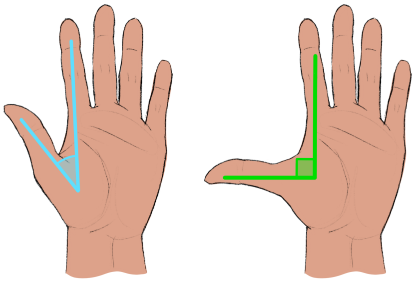
For further reference, a version of this chapter has been published independently [Chatain et al., 2020].



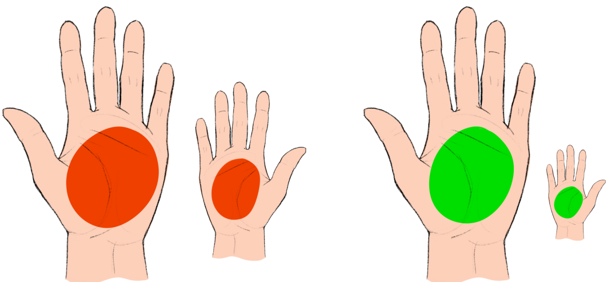
(a) Counting in $\mathbb{N}$



(b) Counting in $\mathbb{Z}/8\mathbb{Z}$



(c) Measuring Angles



(d) Embodying proportions

Figure 3.11: *__DigiGlo__ can be used to support embodied learning of mathematics. For example, our mechanism can support finger counting in $\mathbb{N}$ (a), but also in other spaces (b). Moreover, __DigiGlo__ could be used to offer an on-body implementation of previous embodied learning activities such as The Hidden Village (c, [Nathan and Walkington, 2017]) and the Mathematical Imagery Trainer (d, [Howison et al., 2011]).*

Designing the interaction

Grasping mathematics can be difficult. Often, students struggle to connect mathematical concepts with their own experiences and even believe that math has nothing to do with the real world. To create more concreteness in mathematics education, we focus on the role of the body in learning, and more specifically, embodied interactions for learning derivatives. In this project, we designed an embodied game to teach derivatives, and validated our design with a panel of experts. We then used this prototype to explore different embodied interactions in terms of usability, sense of embodiment, and learning outcomes. In particular, we evaluated different degrees of embodied interactions, and different types of embodied interactions in Virtual Reality. We conclude with insights and recommendations for mathematics education with embodied interactions.

Figure 4.1: *Three different ways of learning derivatives using embodied interaction: direct-embodied interaction on tablet, direct-embodied interaction in Virtual Reality, and enacted interaction in Virtual Reality.*

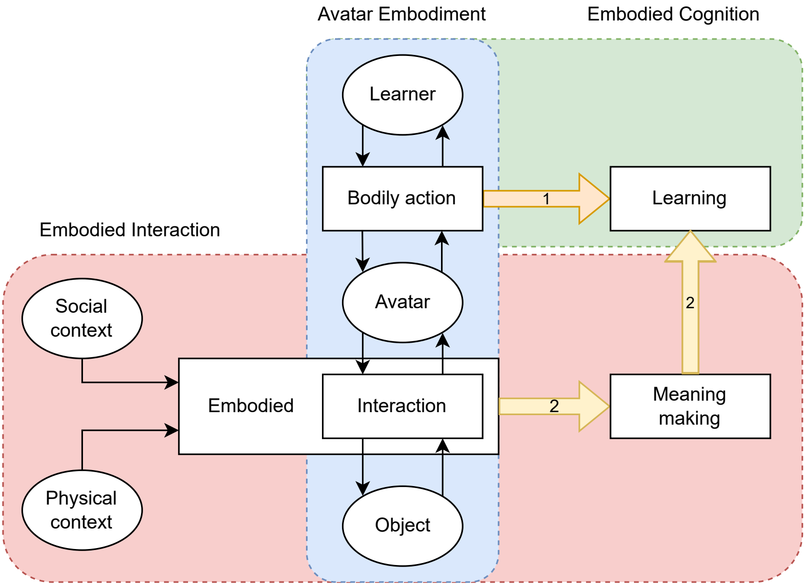


Figure 4.2: *System representation of the embodiment landscape, highlighting two pathways to learning: (1) the path of embodied cognition:* `Bodily action` ⇒ `Learning`*, and (2) the path of embodied interaction:* `Interaction` ⇒ `Meaning making` ⇒ `Learning`.

4.1 Introduction

As described in Chapter 2, embodied interaction and embodied cognition are two different meanings of the term "embodiment" and this distinction is often overlooked. Moreover, despite growing evidence for the benefits of embodied learning and digital experiences, the influence of interaction design choices on embodied learning is under-explored.

Looking back at our embodiment system, we argue that two pathways to embodied learning should be explored (Figure 4.2). First, the direct path from bodily actions to learning, when bodily actions represent or enact mathematical concepts. This is the path most often considered in embodied cognition research. Second, the path informed by the context of the embodied interaction, granted that the meaning of the interaction aligns with the mathematical meaning. Specifically, we argue that a certain interaction technique can be meaningful without being necessarily mathematically meaningful, and that congruence between interaction meaning and mathematical meaning should be investigated in more depth.

In this chapter, we address this issue by exploring the implications of interaction design choices. To do so, we design a game to teach derivatives to high-school students. Through a qualitative study with a panel of experts, we identify the strengths and limitations of such embodied activity. Based on this feedback, we build a improved prototype, which we use to answer our research question. Using different embodiment frameworks, we design several variations of the prototype, addressing different degrees of embodiment [Tran et al., 2017; Johnson-Glenberg et al., 2014], as well as different types of embodiment [Melcer and Isbister, 2016]. Through a quantitative study, we evaluate the impact of these parameters on usability, the sense of avatar embodiment, as well as learning outcomes.

In this chapter, we describe our process, and through this mixed-methods approach [Dingyloudi and Strijbos, 2018], we contribute with design recommendations as well as quantitative evidence for the appropriate degree and type of embodied interaction in order to support embodied cognition.

4.2 Design

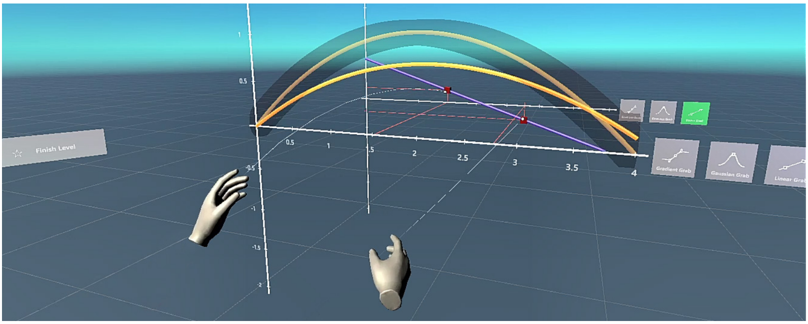


Figure 4.3: *Example of a level from our first prototype, seen from a third-person perspective. The player manipulates the derivative curve (purple, in the back) in order to fit the function curve (yellow, in front) in the target area (grey).*

4.2.1 Initial design

We implemented a Virtual Reality (VR) game supporting exploration and intuition-building of the derivative concept through embodied interaction. For this first prototype, we focused mainly on interaction and level design, from the perspective of embodied cognition [Abrahamson et al., 2020]. As recommended in the *activities* design guidelines [Abrahamson and Lindgren, 2014], we used no symbolic stimuli for this activity and focused on graphical representations. Each level of the game displays two curves (Figure 4.3): one curve represents the function (in front, in yellow), and one curve represents its derivative (in the back, in purple). The level also displays a target curve and a target area related to one of the curves (yellow/purple and grey).

In order to pass the level, the player needs to manipulate one curve to put the other curve in the target area. For example, on Figure 4.3, the player has to manipulate the derivative curve in order to give a bell shape to the function curve. Once the curve is in the target area, the "Finish Level" button turns green and the player can validate their solution. We provide percentage accuracy outcome-feedback computed according to the distance between the player's proposed curve and the target curve [Johnson et al., 2017]. If they are perfectly aligned, this score is 100 %.

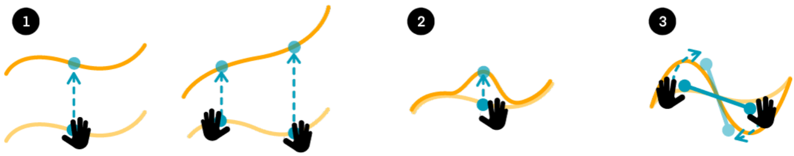


Figure 4.4: *The three grab modes implemented in our prototype. (1) The linear grab mode applies a linear transformation corresponding to the hand movements. (2) The Gaussian grab mode adds a Gaussian shape to the selected node according to the hand movement. (3) The gradient grab mode modifies the local value of the derivative according to the slope between the two hands.*

We explored the interaction space through three embodied interaction modes (Figure 4.4), focusing on congruent gestures [Johnson-Glenberg, 2019]. With the *linear grab* mode, the player grabs the curve with one or two hands, and all the points of the curve are moved along the y-axis, by an offset linearly interpolated between the offsets of each hand. If only one hand is used, the curve is translated along the y-axis. With the *Gaussian grab* mode, the player moves a point of the curve up and down. The neighbor-

Table 4.1: *Design Evaluation: Experts' profiles.*

ID	Age	G.	Expertise	Math. training	Math. affect	Sports	VR
P1	25-29	F	Embodiment and haptic feedback	College (secondary)	High	Avg	Low
P2	25-29	M	Math learning (math graduate)	College (main)	High	High	Avg
P3	25-29	F	Math education (doctoral student)	College (main)	High	Avg	Low
P4	25-29	M	Art and game development	College (secondary)	Low	Low	High
P5	35-39	M	Embodied cognition (mathematics)	College (main)	High	Low	Low
P6	30-34	F	Cognitive sciences (children & adolescents)	High-school	High	High	Low

ing points are moved following a smooth Gaussian shape. Finally, with the *gradient grab* mode, the player manipulates the slope at a specific point by rotating their hands to the desired slope value.

Following the *materials* and *facilitation* guidelines [Abrahamson and Lindgren, 2014], we start by immediate environmental outcome-feedback loops [Johnson et al., 2017]. In the *normal levels*, the curve is updated in real time, as the player manipulates it. To help the students evaluate their understanding we also offer *delayed-feedback levels* where the curve is only updated once the player releases it. This way, the player can use the *normal levels* to explore the relationship between the curves and gain intuition, and is then invited to think deeper during the *delayed-feedback levels* as the interaction is less direct.

Finally, to help the player connect their interaction to numerical values, we added axes for each curve. When the player selects a point, the projected rays of the point towards the x and y axes are displayed in red (Figure 4.3).

4.2.2 Design Improvement

We invited six experts to evaluate our prototype, individually (Table 4.1). Each expert tried the game for 15 minutes and was invited to speak about their experience while trying out the game (*think-aloud* comments). We then conducted a semi-structured interview with each expert where we asked general questions about the game and questions related to their specific expertise. We analysed the transcripts using an inductive thematic approach [Braun and Clarke, 2006], to identify areas of improvement. From the experts' comments, we improved our design in several ways, detailed in the following sub-sections. We discuss our decisions in relation to the design principles for embodied interaction in VR [Johnson-Glenberg, 2019].

Embodied Interaction

Overall, participants found the interaction with the curves intuitive: "It is very self-explanatory, it works, it is nice" (P2), "It was really easy to use, even for inexperienced people" (P1), "The actual interaction felt fine, there wasn't anything unnatural" (P5). However, sometimes is was not clear to them which curve they should interact with, nor which grab mode was activated: "I just was not sure which lines I was able to move, what tool do I have?" (P2). To address these issues, we focused on only one mode of interaction, the gradient grab, and only one curve to manipulate, the function curve.

Regarding the embodied part of the interaction, participants found it enjoyable: "It feels good when I change stuff" (P2), "I felt very comfortable" (P1). P5 found the *normal levels* more enjoyable than the *delayed-feedback levels*: "I just liked moving them and seeing a response. [...] When it was not being updated, it was not like I disliked it, but I did not get that sort of kinesthetic enjoyment out of it". P5 also enjoyed large movements more than restricted movements: "It was a more enjoyable thing to stretch out more". To include this in our design, we implemented the interaction techniques without any restriction on movement amplitude.

Moreover, embodied interaction creates a hands-on experience with mathematical concepts: "If you engage the whole body, you are automatically more engaged [...], just because you have this experience of being there with the curves. You are immediately closer to the topic" (P6), "Because I was really moving the [curve], I appropriated the curves to what I was doing and I learned that there are links" (P4). This way, the mathematical objects are manipulable and perceptible, and, therefore, concrete. However, participants would have preferred an even more direct interaction with the curves: 'I could not do something the way I wanted to, because I could not really 'grab' the curve" (P3), "But to really feel embodiment I would need to really move the things without any distance" (P4). We addressed this by using hand-tracking over controllers "for active, body-based learning" [Johnson-Glenberg, 2019], without any physical distance between the user and the curves, and we included a skin color selection panel.

Finally, the interview with P5, an embodied cognition expert, revealed that an amount of desirable difficulty in the embodied interaction can actually benefit learning: "If you're trying to create a good user interface, then you want to make it seamless, but if you're trying to get people to learn, then it oftentimes helps to throw in some difficulty or something that makes them think". He also mentioned that the *delayed-feedback levels* play in that direction: "For example, the fact that the line was not updating is good for that. Even though, personally, it did not make me feel good, that is not a bad thing". He also suggested several ideas in that area, such as making only certain parts of the curve manipulable, or restricting the movements. Indeed, some difficulty such as lack of feedback can be beneficial to learning [Bjork et al., 2011; Fyfe and Rittle-Johnson, 2017]. Considering desirable difficulties in our prototype, and aligning with the "Use guided exploration" design principle [Johnson-Glenberg, 2019], we reduced the interactability of the curve to a set of specific points, defined per level, and composed of the minimum amount of points necessary to define the curve. However, we decided to stay in alignment with our previous findings and not reduce the movements.

Mathematical Understanding

Overall, our panel of experts agreed that our activity is a novel and interesting approach to mathematics that helped them sharpen their intuition, through exploration: "You can, in a fun way, gain intuition and see" (P2), "It's a cool new dimension that I didn't know before, It was great to see this connection directly" (P3), "It makes it less like a recipe and more like the gradient actually has something to do with how the function looks" (P1), "I gained some sense of quantity of difference: 'If I do about this much to this line, the other line should move by about this much' " (P5). P3, mathematics education expert, mentioned that such an approach could be useful for students: "High school students would benefit from introducing the first derivation to sharpen their intuition, but also people in first years of college to get a different approach than only formulas and rules." We reinforced this exploratory approach by adding a short text at the beginning of the activity, inviting the students to explore the relationship between the two curves. We "minimized text reading" [Johnson-Glenberg, 2019].

To solve the problems, participants used strategies focused on intuition or trial and error: "I could still call up certain intuitions" (P3), "I don't have any tactics. I just like how it feels" (P1), "If something did not work, I would try something else immediately" (P4). *Delayed-feedback levels* invited some participants towards deeper reasoning: "You do not just try things out but rather you have to think about it" (P3), "They were important, because they made me realize my difficulties" (P1), "I could have still done trial and error. It would just take longer and be less satisfactory" (P5), "I gained some intuition, which I then tried to apply on those *delayed-feedback levels*" (P2). We kept this mechanism in our new prototype, adding *delayed-feedback levels* at the end of each section, as a mean to "design in opportunities for reflection" [Johnson-Glenberg, 2019], and align with the need for desirable difficulties previously identified.

Finally, we also identified the need to reconnect our activity with a more formal or traditional form of instruction: "It would be optimal if you connect it with the underlying theories" (P2), "Once the students sharpened their intuition, you can say 'Yes, but what does that mean now?' " (P3), "It should [...] have another kind of learning in the session. [...] You want people to learn the logic, and not guessing" (P4). Having a phase of exploration followed by instruction is a well-known pedagogical pattern, more generally called PS-I for "Problem Solving followed by Instruction", that has shown great potential for mathematics education [Sinha and Kapur, 2021]. This approach relies on three main mechanisms: activation of prior knowledge, awareness of knowledge gap, and recognition of deep features [Loibl et al., 2017]. Our activity seems particularly suitable for this approach: participants can connect to the exercise, as well as identify knowledge gap in the *delayed-feedback levels*. In our final design, we integrated the activity in a PS-I pedagogical pattern by adding an instructional video after the activity.

Interface Design

We identified several issues related to interface design. Several participants mentioned that the grid was difficult to use, and, even difficult to see: "Having clear numbers there would be nice" (P2), "I did not even see the grid" (P6). We improved the readability of the grid: we made the unit graduations more visible, and highlighted the exact values corresponding to the selected points.

The participants also mentioned that the positioning of the curves creates occlusions: "Sometimes you cannot see the second line because of the front line" (P2), "It is a bit unfortunate sometimes that the two graphs were behind one another" (P2). We resolved this issue by placing both curves in the same plane, and adding a mini-display to provide an overview of the level.

Finally, P4, digital artist, mentioned that the visuals should be improved to be more appealing and attractive: "It is always a challenge to make math appealing. [...] Maybe visuals that could be a bit more enjoyable, you could have something more colourful." With this aim in mind, we need to also be mindful about our color choices, as the purple was difficult to see for P3: "I find the purple line at the back difficult to see". We improved our prototype by designing a colorful VR room with windows, plants, and we changed the colors of the curves to yellow and pink.

Virtual Reality

When designing embodied activities in VR, it is important to remember that VR is novel and requires an adjustment period: "Assume every learner is a VR newbie—start slow" [Johnson-Glenberg, 2019]. Adding this on top of some potential math anxiety might also make the experience overwhelming for some users [Lorenzen, 2017]. This was reflected in several comments: "I am struggling way more with the technology than with the task itself, I am just inexperienced" (P1). In particular, P6, who has very little experience with VR and game controllers, felt overwhelmed by the system: "I was too focused on everything that was so new to me. I was also focused on the... what are they called... the controllers. [...] Because [of that], I felt I could not do this".

To some extent, VR can even restrict the users' movements: "Because I do not have a lot of experience with VR, I am very careful when moving because I do not know if I am going to hit anything." (P1), "I just did not want to walk into something." (P5). Beyond the risk of colliding with the real world, the imprecision of the tracking can also impact the experience: "It is pretty difficult to aim easily" (P4), "With keyboard and mouse as input, we are much more precise. And precision is good in some aspect, you can reach 100 % at every level if you are exact" (P2).

To mitigate these issues, we added a tutorial to our activity where participants can explore the VR space and grow trust for the digital boundary appearing as they reach the limit of the space. We also included an interaction tutorial where users can get familiar with hand tracking. Moreover, to improve precision, we selected a pinching gesture, over a grabbing gesture, for curve manipulation.

Finally, all experts agreed that the use of VR benefited their experience and connection with the mathematical content, in particular compared to a screen or a tablet activity: "I think that Virtual Reality is just more similar to reality than a tablet, or a computer screen and a mouse" (P6), "It was really cool to have the whole room and to see the curve in front of you, and not only in the screen. It felt like I was really there with the curve" (P6), "I would prefer [VR] over [a tablet] because there is some feeling here, which is not just like pointing, it is also grabbing" (P5).

4.2.3 Final design

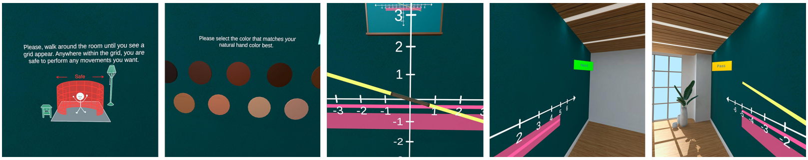


Figure 4.5: *Different steps of the final activity in first person view: Familiarization with* VR *space, skin color selection, level solving, "Next" button to go to the next level, "Pass" button appears after one minute.*

In this final prototype, the game goes as follows: First, the participant explores the VR environment to understand the space and feel safe. Second, the height and the hand size of the participant are calibrated. Third, the participant picks their natural hand color across 12 different tones, and, finally, the activity starts (Figure 4.5).

The activity contains a prompting text to explore the relationship between the yellow and pink curves, as well as a tutorial level with animated hands demonstrating the interaction technique, followed by 21 levels. In each level, the participant manipulates the function curve (yellow) to move the derivative curve (pink) into the target area (pink). The function can be manipulated at specific points (wooden handle). The resulting curve is approximated using constrained cubic splines [Kruger, 2003], for smooth interpolation, and small movements having small effects on the interpolated curve.

After level completion, a "Next" button — positioned above the user, to provide an embodied interaction focused on the feeling body (*Leib*) [Mueller et al., 2018], on the right side, to align with wide-spread interaction paradigms — can be pressed to proceed. One minute into a level, a "Pass" button appears. The levels progress in difficulty, focusing on different topics. Each topic contains several *normal levels* and finish with *delayed-feedback levels*. On a mini-display, the user can keep track of the level and score, which represents how close the manipulable points of derivative are to the target derivative.

Finally, we designed the levels so as to target specific learning goals, summarized in Table 4.2.

Table 4.2: *Learning Goals.*

Core Concepts	
UP	A (strictly) positive derivative reflects an (strictly) increasing function
DOWN	A (strictly) negative derivative reflects an (strictly) decreasing function
FLAT	A null derivative reflects a constant function
SLOPE	There is a link between the local slope of a function and the value of the derivative
VAR	There is a link between the variations of a function and its derivative

Emerging Concepts	
EXTM	At an extremum, the derivative is null
CST	The derivative does not change if the function is shifted by a constant

4.3 Quantitative User Study

After having shown the potential of our prototype for teaching derivatives with embodied interaction, we designed a study to quantitatively answer several research questions:

- How do different embodied interactions compare in terms of usability and resulting manipulations?
- Which embodied interaction brings the greatest sense of embodiment and sense of agency?
- How do different embodied interactions influence learning outcomes?

4.3.1 Embodied Interactions

Addressing our research questions, we consider the degree and type of embodiment. According to the embodiment matrix (Figure 4.6), for the type of embodiment, we compare the conditions direct embodiment on tablet (TAB, left on Figure 4.1) to direct embodiment in VR (DIR, center on Figure 4.1). To compare the type of embodiment, we compare the DIR condition to the *enacted* embodiment in VR condition (ENA, right on Figure 4.1).

The degree of embodiment We compare low (degree 2/tablet) to high (degree 4/VR) embodiment (first row on Figure 4.6). We expect students in the lower embodiment condition to experience lower sensorimotor engagement due to gestures of a smaller amplitude (pointing versus grabbing), partial body engagement, as well as reduced immersion due to a limited coverage of the field of view.

The type of embodiment Similarly, we compare bodily action: In the *direct-embodied* condition, the position of the user represents the derivative, while in the *enacted* condition, the movement of the user represents the derivative.

In the *direct-embodied* interaction, the user holds a proxy of the slope of the curve and manipulates it to influence the derivative. The slope between the user's hands represents the local slope of the curve, that is, the derivative. This approach emphasizes the derivative as a slope. For the *enacted interaction*, the user draws the desired slope by hand. The hand movement thus describes the derivative. This approach emphasizes the derivative as a variation. Both conditions are illustrated on the last column of Figure 4.6.

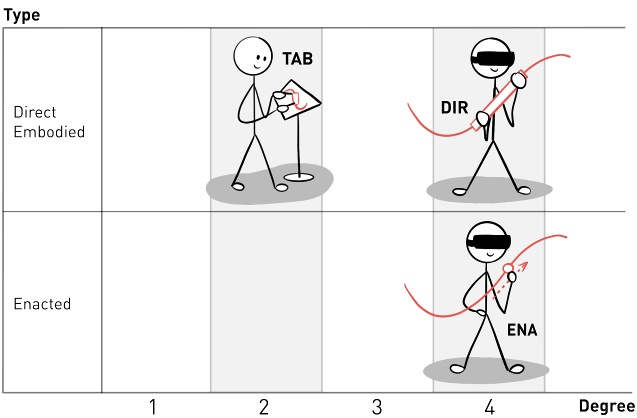


Figure 4.6: *Embodiment matrix. The degree is compared along the horizontal axis, and the type is compared along the vertical axis.*

4.3.2 Demographics

We recruited 40 public high school students, from two different classes taught by the same teacher, who chose to be taught in English. No participant was repeating the class. Two students dropped out, resulting in a final sample of $n = 38$ students with a mean age of $M = 17.6$ $(SD = 0.61)$ and 21 different mother tongues. 19 students identified as male, 18 as female, and 1 as other. The study took place a couple of weeks before the lesson on derivatives and the students knew how to read functions' graphs. The study was conducted in English and at the schools. Due to health regulations, face masks were mandatory at all times. The students received a financial compensation for their participation.

4.3.3 Protocol

We followed an in-between experimental design to avoid carry-over effects and fatigue effects. The study took place during class-time, at the school, in a room large enough to host the VR spaces (2.5 m * 1.7 m each). A preparatory intervention (20 min), and a PS-I intervention (1 h) were conducted on different days, with 1-7 days in between to avoid fatigue effects.

In the preparatory intervention, the students filled out several questionnaires followed by a 10 minutes VR game, "*Elixir*", heavily focused on hand tracking [Magnopus, 2020]. Through the questionnaires we obtained information about prerequisites, demographics, math anxiety [Hopko et al., 2003], and body awareness [Shields et al., 1989].

After the preparatory intervention, we randomly assigned students to the conditions and balanced for prior knowledge, math grade, gender, VR experience, math anxiety, and body awareness. 13 participants were assigned to the TAB condition (7 male, 6 female), 12 to the DIR condition (6 male, 6 female), and 13 to the ENA condition (6 male, 6 female, 1 other).

During the PS-I intervention, the students filled in a Simulator Sickness Questionnaire (SSQ) [Kennedy et al., 1993]. Subsequently, they performed the derivative activity, either on tablet or in VR according to their condition. Then they filled in the same SSQ, and a System Usability Scale (SUS) questionnaire [Brooke and others, 1996]. The participants in the DIR and ENA conditions filled in a Sense of Embodiment questionnaire, adapted to focus on the hands [Roth and Latoschik, 2019]. All participants filled in a questionnaire about felt agency on the mathematical objects adapted from an Avatar

Embodiment questionnaire [Gonzalez-Franco and Peck, 2018], and their usage of the tool. After this, the students watched an instruction video about derivatives, using the same color scheme as the exploratory activity, and recorded by an English native speaker. Then the students took a 5 minutes break where they could read comics, in order to avoid fatigue effects. Finally, the participants solved a post-test evaluating their understanding of derivatives, and a selection of questions on first derivatives from the Calculus Concept Inventory (CCI) [Epstein, 2007]. The post-test focused on specific properties of the derivative and was presented in a visual style, while the CCI required to combine several properties and resembled classical math problems.

The tablet intervention was conducted on Apple iPad 5th Gen 32GB 9.7", and the VR interventions on Oculus Quest 2. During the activity, we logged general information about the participants (height, hand size, skin color), time to level completion, level completion or skipping, and manipulations the mathematical objects.

The implementation and study design were validated through a pilot study with 19 high-school students.

4.3.4 Questionnaire design

In order to assess learning, we created our own questionnaires. In this section, we describe our process. As our activity focused on graphical representations of functions and derivatives, and did not target symbolical expressions and formulas, we do not address these in the questionnaires. In the post-intervention questionnaires, we used a color scheme similar to the one used in the activity.

Prerequisites questionnaire

During the preparatory intervention, the participants filled in a prerequisites questionnaire. The goal of this questionnaire was to assess whether or not the participants had all the required prerequisites to understand the new concepts. We expect a ceiling effect with this questionnaire, as lower scores indicate that the students do not have all the necessary prior knowledge to understand the new materials. As these questionnaires were filled in a week before the second intervention, the effect of activation of prior knowledge is mitigated.

We identified several prerequisites for each of learning goals, summarized in Table 4.3.

Table 4.3: *Prerequisites for each learning goal.*

Goal	Prerequisites
UP	Given the graph of f, the student can identify where f is positive
	Given the graph of f, the student can identify where f is increasing
DOWN	Given the graph of f, the student can identify where f is negative
	Given the graph of f, the student can identify where f is decreasing
FLAT	Given the graph of f, the student can identify where f is null
	Given the graph of f, the student can identify where f is flat
SLOPE	Given a vector, the student can read its coordinates
VAR	Given the graph of f, the student can identify f's monotonicity properties
	Given x and the graph of f, the student can identify the value of $f(x)$
EXTM	Given the graph of f, the student can identify where f is null
	Given the graph of f, the student can identify its extremums
CST	Given x and the graph of f, the student can identify the value of $f(x)$

To address all of these aspects, we designed questions about identifying the sign of a function on certain intervals, identifying the monotonicity properties of a function, reading the coordinates of a vector, reading the image of certain abscissas under a function, and identifying the extremums of a function. For each of these aspects, we check the prerequisite in different configurations. For example, on the graph presented Figure 4.7, we ask for the value of $f(6)$, $f(-2)$, and $f(-5)$.

Post-activity questionnaire

At the end of the Problem Solving followed by Instruction (PS-I) intervention, we evaluated learning on three aspects: isomorphic problem solving, transfer, and conceptual understanding. In this section, we describe our process and the resulting questionnaires.

Isomorphic Problem Solving questionnaire In the isomorphic questionnaires, we designed questions similar to the problems they had to solve in

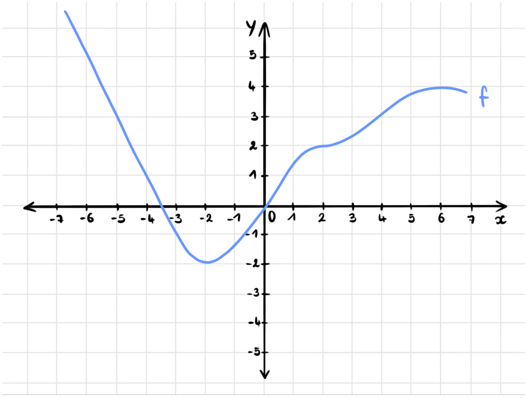


Figure 4.7: *Example graph in the prerequisite questionnaire.*

the embodied activity. Concretely, given a target derivative graph, the students should draw the desired slope of the function at specific points (Figure 4.8).

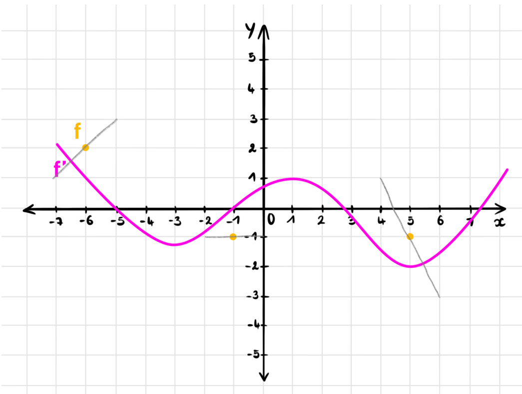


Figure 4.8: *Example solution for a task in the isomorphic problem solving questionnaire.*

We started with a warm up exercise, with a derivative flat over three intervals, and either negative, null, or positive.

In the next exercises, we covered 9 scenarios, covering all combinations where the function is increasing, decreasing, or flat, while the derivative is increasing, decreasing, or flat.

This questionnaire addresses the following learning goals: UP, DOWN, FLAT, and SLOPE.

Transfer questionnaires With the transfer questionnaire, we addressed several kinds of transfer: direction, continuity, abstraction, and dimension.

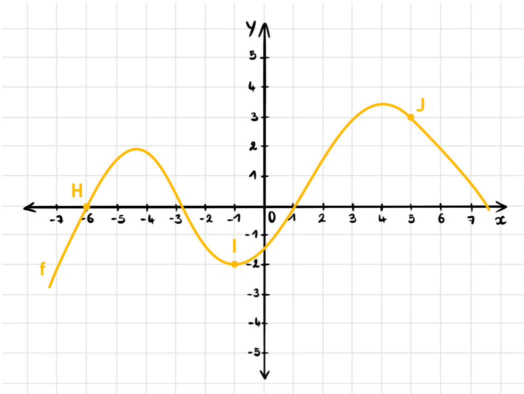


Figure 4.9: *Example task in the "direction" transfer questionnaire. The students have to determine the value of the derivative at points H, I, and J.*

To address transfer to another direction, we reversed the task of the isomorphic questionnaire. In the isomorphic questionnaire, students had to draw the desired slope of a function from the value of its derivative. In this first transfer questionnaire, we check if the students can solve the problem in the reverse direction, that is: from the function, can the students determine the value of the derivative in certain points (Figure 4.9)? We address all 9 combinations where the function is positive, negative, or null, while the derivative is positive, negative, of null.

This questionnaire addresses the following learning goals: UP, DOWN, FLAT, and SLOPE.

As all questionnaires so far prompted the students about values at specific points, we addressed transfer to continuous situations by designing two drawing tasks. In the first task, the students have to draw, given the graph of a function, the continuous graph of its derivative (Figure 4.10).

In the second task, the students have to draw the shape of the function, given its derivative (Figure 4.11).

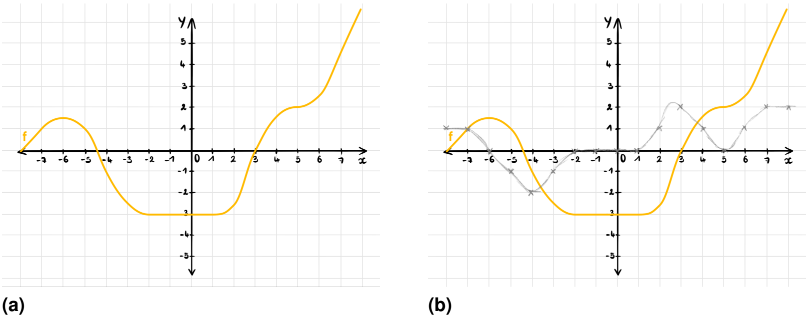


Figure 4.10: *First task of the "continuity" transfer questionnaire (a) and example solution (b).*

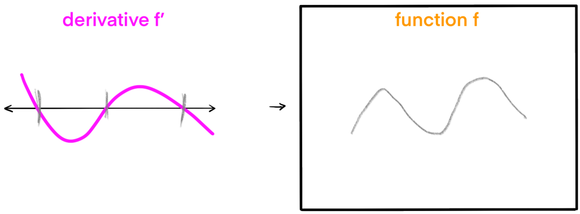


Figure 4.11: *Example solution for a task in the second task of the "continuity" transfer questionnaire.*

These questionnaires address the following learning goals: UP, DOWN, FLAT, SLOPE, and VAR.

Another important aspect of transfer, in the context of this doctoral work, is abstraction. And in particular, abstraction towards symbolical representations. To address this question, we provide the students with a brief explanation of sign tables, and ask them, given the sign table of the derivative, to draw a possible graph of the corresponding function (Figure 4.12).

This questionnaire addresses the following learning goals: UP, DOWN, FLAT, SLOPE, and VAR.

Finally, we evaluate whether or not students can extrapolate their understanding of derivatives to higher dimensions. After a brief explanation of derivatives along a specific axis, the students are asked to indicate the proper derivative at a specific point on a 2D surface (Figure 4.13). The same surface

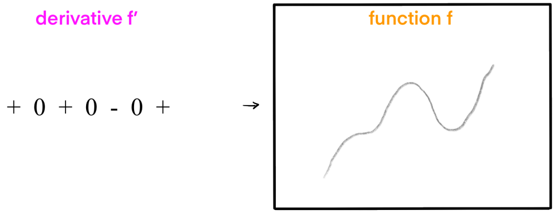


Figure 4.12: *Example solution for a task in the "abstraction" transfer questionnaire.*

is always represented from two different perspectives: one on the x-axis, and one of the y-axis.

This questionnaire addresses the following learning goals: UP, DOWN, FLAT, and SLOPE.

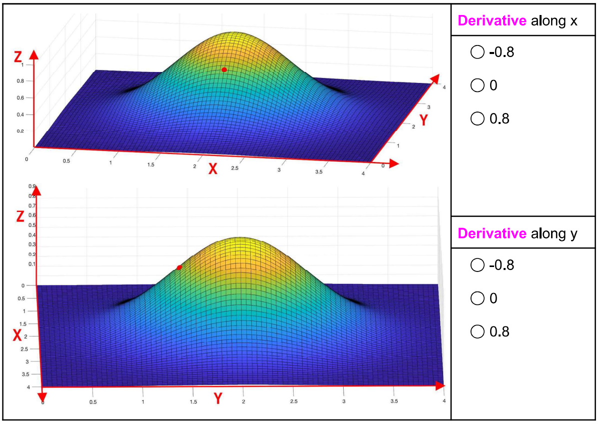


Figure 4.13: *Example task in the "dimension" transfer questionnaire.*

Conceptual Understanding questionnaire The last aspect we wanted to address is conceptual understanding. In this questionnaire, we start by a general question to assess whether or not the students understood the main concept, and then focused on aspects that can be extrapolated from the activity, but are not explicitly mentioned. The questions of the conceptual understanding questionnaire are:

1. How would you explain what a derivative is to your friend who doesn't know?
2. What do you think the value of the derivative is at the maximum of a function? Please justify your answer.

3.1. Do you think that two different functions can have the same derivative? Please justify your answer.

3.2. Do you think that it is possible to trace the graph of a function from the graph of its derivative? Please justify your answer.

This questionnaire addresses specifically the learning goals related to the emerging concepts: EXTM, and CST.

4.3.5 Results and analysis

The degrees (TAB and DIR conditions) and types (DIR and ENA conditions) of embodiment were compared using independent Welch t-test or Yuen test [Yuen, 1974] following the results of the Shapiro-Wilk normality assumptions check (Table 4.4).

Usability and Resulting Manipulations

There was a significant difference in duration in degree of embodiment with a very large effect size. In average, it took $M = 9.27$ ($SD = 2.00$) minutes for the participants in the TAB condition to solve all the levels, compared to $M = 16.42$ ($SD = 3.45$) in the DIR condition. Similarly, a significant difference was found concerning duration in types of embodiment with a large effect size. In average, participants in the ENA condition took $M = 23.22$ ($SD = 3.45$) minutes to complete all the levels.

Regarding the number of manipulations with the curve, we found no significant difference across the degrees of embodiment with a medium effect size. However, we found a significant difference between the types of embodiment with a very large effect size. Participants in the DIR condition interacted $M = 133$ ($SD = 25$) times in average, while participants in the ENA condition interacted $M = 246$ ($SD = 83$) times in average. In the *direct-embodied* conditions, the participants usually grabbed the handle and adjusted until satisfaction. On the other hand, people in the *enacted* condition often released the knob and tried again. This difference also explains the duration difference.

Table 4.4: *Inventory of the t-tests results. A result was considered significant ($*$) when $p < 0.05$ and almost significant when $p < 0.10$ ($\cdot$). Cohen's d and Cohen's U_3 effect sizes are reported [Valentine and Cooper, 2003; Hanel and Mehler, 2019]. Cohen's U_3 represents distribution overlap and is the percentage of participants in the lower-mean condition scoring lower than the mean score of the participants in the higher-mean condition.*

	Degree: TAB and DIR					**Type: DIR and ENA**				
Dependent variable	df	t	p	d	U_3	df	t	p	d	U_3
Duration	17.38	−6.27	< **0.001**$*$	2.6	100%	18.10	−3.19	**0.005**$*$	1.25	100%
# of manipulations	18.23	1.69	0.11	0.65	83%	14.38	−4.64	< **0.001**$*$	1.79	100%
SUS score	22.95	−0.10	0.92	0.04	38%	20.43	1.23	0.23	0.50	67%
Delta SSQ score	14.86	1.23	0.24	0.47	77%	10.82	0.90	0.39	0.33	83%
Total hand movement						10.02	1.56	**0.001**$*$	1.83	92%
Avg hand movement						7.91	4.86	**0.001**$*$	1.98	92%
Avg amplitude						20.36	16.72	< **0.001**$*$	6.57	100%
Body ownership						20.87	−0.88	0.38	0.35	67%
Body agency						22.87	0.24	0.81	0.1	46%
Body change						22.02	−0.01	0.99	0.06	58%
Curve agency	20.5	−0.08	0.94	0.03	46%	21.40	1.66	0.11	0.65	54%
Learning Post-test	9.52	1.45	0.18	0.58	50%	11.79	0.27	0.79	0.10	54%
CCI	22.70	0.54	0.59	0.22	75%	22.97	1.76	**0.09**$\cdot$	0.70	69%

Regarding usability, we computed the SUS scores for each condition: TAB scored 68 ($SD = 12$), DIR scored 69 ($SD = 12$) and ENA scored 62 ($SD = 16$). We noticed that the first question of the SUS "I think that I would like to use this system frequently" scored rather low ($M = 2.83$, $SD = 1.17$) because students did not necessarily want to study math frequently altogether. Therefore we should refrain from comparing these scores to general SUS scores. Comparing the degrees of embodiment, we expected the DIR condition to be less usable than the TAB condition because of the cumbersomeness of the VR hardware, and the limited accuracy of the hand tracking. However, the t-test was not significant, therefore we could not reject the null hypothesis of no effect. Moreover, the effect size was very small. When comparing the types of embodiment, we expected the ENA condition to be less usable than the DIR condition as the curve should be read left to right but enacting the slope in that direction with the left hand would cover the slope. Once again, the t-test was not significant, however the effect size was medium. This suggests that the ENA condition was slightly less usable than the DIR condition. Regarding the ENA condition in particular, we ran a Pearson correlation test and found no evidence of correlation between the percentage of left hand usage and the reported usability ($r = 0.15$, $p = 0.39$).

As the different degrees of embodiment use different technologies, we expected the DIR condition to create more simulator sickness than the TAB condition. We found no significant difference in delta SSQ scores between TAB and DIR, with a medium effect size. As expected, we found no significant difference between the DIR and ENA delta SSQ scores, with a small effect size.

In conclusion, regarding manipulations and usability, there is no counter indication against a higher degree of embodiment, even though it uses a more cumbersome technology. The only drawback is the significantly-longer duration of the activity. Regarding the type of embodiment, an *enacted* approach is more time-consuming, generates more superfluous manipulations, and might be less usable. Therefore, a *direct-embodied* approach should be preferred.

Sense of Embodiment and Curve Agency

First, we evaluated whether there are movement differences between the types of embodiment. We found that participants in the DIR condition moved their hands more than the participants in the ENA condition. We also found a significant difference in movement per manipulation, and in average amplitude.

With regards to the Sense of Embodiment across the types of embodiment, we found no significant difference for the sense of body ownership, the sense of body agency, and the sense of body change.

Concerning the sense of felt agency on the function curve, we found no significant differences across the different degrees of embodiment, nor across the different types of embodiment. However, we expected participants in the ENA condition to feel less agency on the curve, as the interaction is slightly more indirect, and the low p-value suggests that such effect might be identified with more participants.

We also looked into the impact of body awareness on sense of embodiment and sense of curve agency. Using Pearson's correlation factor, we did not find evidence for correlation between body awareness and sense of body agency ($r = -0.28$, $p = 0.18$), sense of body change ($r = -0.13$, $p = 0.54$), and sense of curve agency ($r = -0.03$, $p = 0.88$). However, we found an almost significant correlation between body awareness and sense of body ownership ($r = -0.38$, $p = 0.06$), meaning that participants with higher body awareness felt less body ownership in the VR conditions.

In conclusion, the *direct-embodied* interaction generated more movement and more amplitude per manipulation than the *enacted* approach. However, this did not translate into a higher sense of embodiment. We also did not find any differences in curve agency across degrees of embodiment, nor types of embodiment. We would therefore recommend favoring a *direct-embodied* approach if an emphasis on movement is desired. We would also advise to be particularly careful on avatar personalisation in VR as higher body awareness led to less body ownership.

Learning and Concept Inventory

The prerequisites scores were very high ($M = 85\%$, $SD = 11.0$), especially for reading the graph of a function ($M = 95\%$, $SD = 16.4$), reading the sign of a function graph ($M = 92\%$, $SD = 17.2$), and reading vector coordinates ($M = 89\%$, $SD = 22.0$). Students scored lower on the questions about reading local maximum and minimum ($M = 64\%$, $SD = 22.3$) but this is less primordial in our activity. There was no significant difference in prerequisite scores between the degrees of embodiment ($t(15.68) = 1.26$, $p = 0.23$), nor between the types of embodiment ($t(12.65) = 0.20$, $p = 0.85$).

Regarding learning, we found no significant difference in post-test scores across degrees of embodiment and types of embodiment. The effect size of the type of embodiment is very small, suggesting that we would not observe an effect with more participants. We then compared the CCI results, and found no significant differences across degrees of embodiment and an almost significant difference with medium to large effect size across types of embodiment in favor of the DIR condition. We also ran a Pearson correlation test using data from the DIR and ENA conditions, and found no correlation between the average amplitude of movement and the post-test scores ($r = 0.17$, $p = 0.44$), nor the CCI scores ($r = 0.08$, $p = 0.72$).

Concerning the number of successfully completed levels, a Fisher exact test yielded no significant difference between the TAB and DIR condition ($p = 0.46$), but an almost significant difference between the DIR and ENA condition ($p = 0.08$, $M_{DIR} = 21.58$, $M_{ENA} = 19.15$), meaning the ENA participants skipped more levels than the DIR participants.

Finally, we carried out a multiple regression to investigate the role of math grade, prerequisites, math anxiety, and body awareness in the final post-test and CCI scores. For the experimental condition, we used a contrast comparing the degree of embodiment (TAB and DIR), as well as the type of embodiment (DIR and ENA). For the post-test score, the results of the regression indicated that the model explained 34% of variance and significantly reflected the underlying data ($F(6,30) = 2.57$, $p = 0.04$). Math grade was the only predictor contributing significantly to the model ($B = 10.00$, $p = 0.035$). In particular, the degree of embodiment ($B = 5.13$, $p = 0.18$) and the type of embodiment ($B = 4.18$, $p = 0.27$) did not contribute significantly. Looking at CCI, the model explained 48% of the variance and significantly reflected the underlying data ($F(6,30) = 4.49$, $p = 0.002$). Again, math grade was the only significant predictor ($B = 0.79$, $p = 0.040$), while the degree and the type of embodiment were not (resp. $B = 0.24$, $p = 0.43$ and $B = 0.41$, $p = 0.19$).

In conclusion, we found no differences in learning across different degrees of embodiment. This might mean either one of two things: there is no effect of the degree of embodiment on learning, or this effect is counterbalanced by the cumbersomeness of VR. Regarding the type of embodiment, the *enacted* approach resulted in worse learning, and a higher quitting rate. Therefore, we would recommend against an *enacted* approach except if the topic at hand requires it. It is also important to note that the math grade was the only significant predictor of the post test scores.

4.4 Discussion

Although mathematics is considered inherently abstract, mathematics learning can benefit from initial concrete examples and representations [Trninic et al., 2020; Fyfe et al., 2014; Carbonneau et al., 2013]. Moreover, students learn mathematics for different reasons. While some students might decide to dedicate their career to the topic, others will only use their mathematics skills as a tool in other contexts: focusing the lesson solely on abstract symbols and formalism does not reflect such individual differences. Similarly, students suffering from math anxiety can benefit from hands-on experiences [Chen, 2019].

With our work, we offer an embodied activity to discover and explore concrete derivatives, while gaining intuition. From our design process and empirical results, we present several aspects to consider when designing embodied interaction for learning mathematics. Indeed, although VR is promising when it comes to highly embodied interaction, such technology is also time consuming and spatially cumbersome. As designers, we ought to make the experience worth the logistics, and go beyond the increased motivation tied to the technology [Kavanagh et al., 2017].

4.4.1 Take-away messages

First, although VR can indeed increase the sense of embodiment and movement amplitude, this is not always automatic. For example, participants with less VR experience might feel afraid and reduce their movements. Moreover, selecting a more indirect form of interaction might result in reduced sense of agency, as well as less movements and smaller amplitude. We recommend preceding the VR activity by an exploration phase where students discover the virtual space and its limits, as well as the interaction possibilities. Moreover, we recommend favoring more direct forms of interaction, and, if using hand tracking, being mindful of expectations students bring from the real world (Chapter 3 or [Chatain et al., 2020]).

Second, we ought to consider the role of precision in our activity. For example, in an activity with percentage accuracy outcome-feedback such as a score [Johnson et al., 2017], accuracy is of importance, and picking a less precise gesture, such as a grabbing, over a more precise gesture, such as pinching, will increase unnecessary frustration. This aspect goes even further: While high-achieving students will rather focus on the general shape of a graph, other students put a strong emphasis on accuracy when gesturing function representations [Gerofsky, 2011]. We recommend designing interaction matching the precision required by the activity, but also by the target audience.

Third, when designing interaction for learning, we recommend acknowledging the discrepancy between a good interaction from a usability perspective, and a good interaction from a learning perspective, and, in particular, thinking in terms of desirable difficulties [Bjork et al., 2011]. As we saw, delaying visual feedback can create opportunities for reflection, and knowledge gap awareness [Johnson et al., 2017; Johnson-Glenberg, 2019]. Moreover, focusing the interaction on specific areas of the problem at hand can help the student focus on the critical aspects.

Finally, we want to emphasize the importance of designing embodied interaction not for physical bodies, but for feeling bodies [Mueller et al., 2018]. Although we did not focus our study on this aspect, we did notice that these design choices were particularly enjoyed by the students. For example, following the recommendation of Mueller et al., we placed the button to finish a level on the top right of the user [Mueller et al., 2018]. As a result, students soon turned this interaction into a "high-five" motion, and, we believed, appreciated their achievement at an embodied level.

4.4.2 Two pathways to learning

In Chapter 2, we presented our interdisciplinary framework (Figure 2.10), in which we distinguish bodily actions and interaction. This distinction is important and can be used to interpret our empirical results. In this work, we highlighted different types of embodiment, in particular *direct-embodied*, and *enacted* [Melcer and Isbister, 2016; Ottmar et al., 2019]. While *direct-embodied* interaction focuses on the body as "the primary constituent of cognition", an *enacted* approach emphasizes "learning by physically doing". However, in the case of derivatives, the implications go further. As we designed the *direct-embodied* condition, we used the body position to represent the derivative, and therefore, in this condition, the bodily actions highlight derivatives as slopes. In contrast, in our *enacted* condition, the body movement represents the derivatives and, as a result, the bodily actions emphasize derivatives as variation rates. However, these bodily actions also result in and are directed by interaction. This interaction happens within a certain context in which it gains meaning. In our activity, this interaction takes place at specific positions on the curve, with no notion of spatial or temporal progression, and therefore focuses on the slope of the presented curve.

In conclusion, in the *direct-embodied* condition, the interaction meaning is congruent with the mathematical meaning highlighted by the underlying bodily actions, while it is not the case for the *enacted* condition. In our initial context (Figure 4.2), in the *direct-embodied* case, the meaning highlighted by the interaction is congruent to the one highlighted by the bodily actions, and therefore supports two pathways to learning, represented in our model by `Bodily action` ⇒ `Learning` and `Interaction` ⇒ `Meaning making` ⇒ `Learning`. In contrast, in the *enacted* approach, the two paths are not congruent and result in poorer learning outcomes. Therefore, when it comes to the type of embodiment, we recommend considering the mathematical meaning highlighted by the corresponding bodily actions, and designing the interaction and its context as to be congruent with this meaning.

4.4.3 Limitations and Future Work

The main limitation of the work presented in this chapter is the sample size of the quantitative study. Although this is not an issue for the first two research questions as the expected effect sizes are rather large, it can be an issue for the question on learning outcomes as expected effect sizes are smaller. In particular, the effect sizes regarding the effect of the degree of embodiment on learning outcomes are small to medium, in favor of the weaker embodiment, suggesting that further investigation is necessary. We want to pursue this question with a large-scale study based on our design. Moreover, the learning assessment happened directly after the study: assessments over an extended period of time should be used to address medium and long term effects.

To inform our design, we invited several experts. P3, in particular, has teaching experience in mathematics, but is not an experienced teacher. Inviting a mathematics teacher as well as high-school students from the first step of the design might have revealed interesting findings. The latter was not possible at the time of the design, due to corona-related restrictions.

Another concern is the fatigue effect of the quantitative study as the participants had to fill in several questionnaires. However, as we included a break before the learning outcome questionnaires, we believe that this effect is mitigated.

Similarly, as VR is still a novel technology for most people, some of them might feel anxious when participating in the activity, as was indeed the case in the qualitative study. Conversely, VR might feel exciting for some participants and generate a positive novelty effect [Huang, 2020]. However, we mitigated these effects for the quantitative study by including a preliminary activity where the participants could discover the technology. This was particularly useful as only 3 participants already had VR experience.

Finally, all participants wore a face mask due to the local health regulations, increasing the discomfort of the VR condition. However, they might have already been used to wearing a mask.

Regarding future work, there are several main directions left to explore: First, evaluate whether our recommendations generalize to other topics; Second, evaluate whether they generalize to embodiment of different natures, for example temporal instead of spatial; Third, evaluate in more details how the degrees and types of embodiment influence conceptual understanding, and, in particular, the gestures used to communicate understanding; Finally, evaluate the relation between the design of the embodied interaction and the learning strategies of the users, accounting for individual preferences for gestured graphs [Gerofsky, 2011].

4.5 Conclusion

In this work, we implemented an activity to help high school students build intuition about derivatives. First, we validated our prototype with a panel of experts, and drew conclusions about embodied interaction design; First, although VR is good for embodiment, it requires an adjustment period and can restrict the user's movements. Second, embodied interaction with curves is intuitive and enjoyable, and creates a hands-on experience. Moreover, direct interaction is favored over indirect interaction, although desirable difficulty in interaction can actually benefit learning. Finally, embodied activities offer a novel approach to mathematics and helps building mathematical intuition. However, such activity should be reconnected to formal instruction in order to be truly beneficial to the students.

We then used our validated prototype to compare different degrees of embodiment (weak embodiment on tablet and strong embodiment in VR), and different types of embodiment (*direct-embodied* and *enacted*). Our results show that even though VR technology is more cumbersome and more time consuming, it does not significantly reduce the usability of the prototype nor increases simulator sickness. Moreover, we show that participants using a more indirect interaction, the *enacted* interaction, tend to give up more often and learn less than participants using a more direct interaction. Finally, we did not find differences in learning outcomes across different degrees of embodiment, suggesting that Virtual Reality is not necessary for a successful embodied design.

This work contributes to our research question in two ways. First, it highlights the importance of considering both embodied interaction and embodied cognition perspectives when designing embodied learning activities. Specifically, this framework presents two possible pathways to learning, influenced by design choices. Second, our work offers design recommendations informed by qualitative as well as quantitative results.

For further reference, a version of this chapter has been published independently [Chatain et al., 2022].

Designing the context of the interaction

Abstract mathematics can be difficult to grasp, in part because it relies on symbols and formalisms that are powerful yet meaningless to novices unless grounded in concreteness. Although a wide corpus of research focuses on concreteness in mathematics education, the notion of concreteness can be apprehended in various ways and it is not yet clear which specific aspects of concreteness help the learners. In this chapter, we explore embodiment as a form of concreteness to ground abstract mathematics. First, we designed and evaluated an embodied learning activity on graph theory. Through a user study with 89 participants, we then compared three approaches: abstraction, manipulated concreteness, and embodied concreteness. Our results show that, compared to abstraction, both forms of concreteness increase learners' perceived attention, confidence, and satisfaction. However, only embodied concreteness increases perceived relevance and grounding. Moreover, unlike manipulated concreteness, embodied concreteness does not impair learning outcomes nor transfer abilities.

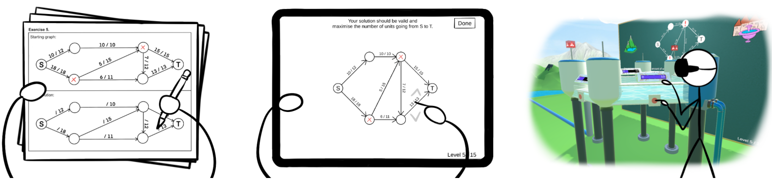


Figure 5.1: *Three approaches: abstraction on paper (*ABST*), manipulated concreteness on tablet (*MNPL*), and embodied concreteness in Virtual Reality (*EMBD*).*

5.1 Introduction

In Chapter 4, we explored the role of interaction in learning mathematics. In this chapter, we consider the context of said interaction. Specifically, we focus on the concreteness of the context of the interaction, and how it can be used to ground more abstract mathematical concepts.

As described in Chapter 2, mathematics can be difficult to grasp for many students. This is, in part, due to the fact that some of the most powerful aspects of mathematics rely on abstract symbols and formalisms that have no meaning, unless grounded in concreteness and provided with an interpretation [Harnad, 1990; Glenberg et al., 2012; Weir, 2011; Nathan, 2021].

But when we talk about abstract mathematics, what are we really talking about? When a mathematician claims that she loves abstraction, while a student protests that he dislikes mathematics because "it is too abstract", are they really talking about the same thing? Similarly, when said student wishes mathematics were more concrete, is a concrete example truly the solution?

There is a verbal dispute in the field of concreteness for mathematics education. Although concreteness has been widely explored as a means to ground abstract mathematics, experts do not always align on their usage of the terms "concrete" and "abstract". For example, looking at only two papers in the field [Fyfe et al., 2014; Pouw et al., 2014], we found that the word "concrete" was associated with a wide range of terms: meaningful, familiar, well-understood, physical, grounded, pictural, perceptual, enactive, real-life, context-specific, and informal. In contrast, the word "abstract" was linked to: general, structural, portable, symbolic, vague, schematic.

What makes a good concrete example? Is it meaningfulness or physicality? Moreover, what is there to appreciate in abstraction? Is it generality or vagueness?

In this paper, we describe different kinds of concreteness, and offer an embodied perspective on the matter. Indeed, both embodied cognition and embodied interaction theories highlight the major role of users' bodies in meaning-making processes and grounding abstract concepts in the real world [Nathan, 2021; Dourish, 2004; Spiel, 2021]. To illustrate the grounding capabilities of embodied concreteness in mathematics education, we designed and implemented an embodied activity to teach graph theory. We then used our activity in a user study to demonstrate the effect of different kinds of concreteness on motivation and learning outcomes. Specifically, we compared three approaches: abstraction, manipulated concreteness, and embodied concreteness.

Our paper illustrates the importance of rigorously defining concreteness and contributes with empirical evidence in favor of embodied concreteness for grounding abstract mathematics.

5.2 Conceptualization of Embodied Concreteness

In this section, we discuss the role of concretenesses in learning abstract mathematics, and describe embodied approaches in this context.

5.2.1 Learning by grounding in concreteness

Learning abstract mathematics is difficult, in particular as, to novices, the related concepts and meaningless conventional symbolic systems can be difficult to grasp [Harnad, 1990; Glenberg et al., 2012]. One way of addressing this issue is by grounding mathematics in concreteness. As described by [Nathan, 2021]:

> [Grounding is the process of mapping] novel ideas and symbols to modality-specific experiences that are personally meaningful.

Specifically, mathematics relies on formalisms, which can be defined as [Nathan, 2012]:

> [Formalisms are] specialized representational forms that use heavily regulated notational systems with no inherent meaning except those that are established by convention to convey concepts and relations with a high degree of specificity.

One way to give meaning to these systems is to reconnect them to concrete instances that give them relevance. In particular, through grounding, a mapping is "formed between an idea or symbol, and a more concrete referent, such as an object, movement or event in the world - as well as mental reenactment of these experiences - in service of meaning-making" [Nathan, 2021; De Vega et al., 2008].

Concretenesses

Before going any further, we ought to define the term "concreteness". Indeed, although concreteness is often discussed in mathematics education,

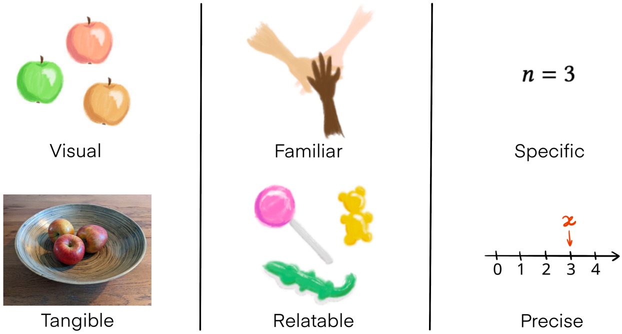


Figure 5.2: *Different concrete representations of the number* 3.

experts do not always align on their definition of concreteness. Identifying such verbal disputes is crucial as they can be tools for progress [Chalmers, 2011]. In this section, we highlight some of the main definitions of "concreteness" in the field of mathematics education. Several examples can also be found on Figure 5.2, and the different definitions considered are presented on Figure 5.3, Figure 5.4, and Figure 5.5.

First, an element can be thought as concrete if it can be touched, felt, smelt, kicked: if it can be *sensed* [Wilensky, 1991]. In that sense, a flower is more concrete than intelligence. This aspect can be influenced by technology, as certain elements can be made visible, for example light paths [Furió et al., 2017], or tangible, for example chemical forces [Müller, 2022]. This concreteness is also influenced by the learners' bodies, as they are central to sensory perception [Keehner and Fischer, 2012; Spiel, 2021].

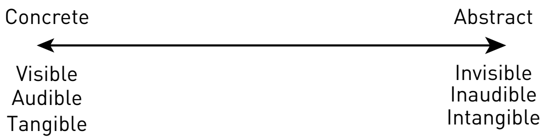


Figure 5.3: *Concrete as Sensed.*

Second, in fields such as mathematics and computer science, an element is often qualified as more concrete if it is more *specific*, constrained, precise, as opposed to general, overarching, and reusable [Wilensky, 1991]. For example, the sequence "{1, 2, 3}, List<int>, List<T>" evolves from more concrete to more abstract. This concreteness depends solely on the element itself and its context.

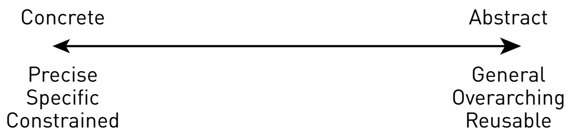


Figure 5.4: *Concrete as Specific.*

Finally, Dewey contrasts a concrete element that can be "readily apprehended by itself" to a more abstract one that can be "grasped only by first calling to mind more familiar things and then tracing out connections between them and what we do not understand" [Dewey, 1910]. Here, an element is more concrete if it is more *relatable*, familiar, or imaginable, and abstract if it is unrelatable, unfamiliar, or unimaginable. With this definition, concreteness is not a property of the element alone, but rather a property of the element as perceived by the learner [Wilensky, 1991]. For example, Papert explains how a gears mechanism was a "comfortable friend" that helped him grasp the "otherwise abstract" concept of equations [Papert, 1980]. From this perspective, learning is a process of concretion, where one grows connections with an abstract element until it is relatable, and in that sense, concrete [Wilensky, 1991].

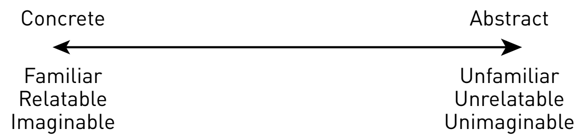


Figure 5.5: *Concrete as Relatable.*

This distinction is important as not all concretenesses align. For example, the mathematical expression $2x - 4 = 0$ is abstract as it has no smell, and cannot be touched. However, for a mathematician, it can be very concrete as it is relatable and familiar. Working with this distinction is important to clearly identify the specific aspects of concreteness that are beneficial for grounding and, therefore, impact learning.

In this work, we consider different kinds of concreteness. Moreover, we do not consider concreteness and abstraction as categories separating elements into two sets, but as relative concepts discriminating elements along a spectrum. Therefore, in this work, the words “concrete” and “abstract” stand for “more concrete” and “more abstract”. Moreover, in our concreteness study, we define our conditions from the most abstract condition, to which we add elements of concreteness. Therefore we refer to the most abstract condition as “abstract” and the other conditions as “concrete”.

Grounding in concreteness

Several ways of grounding mathematics in concreteness have been explored, with mixed results. Kaminski et al. showed that using concrete, specific, relatable examples, as opposed to using abstract representations, is detrimental to transfer of knowledge [Kaminski et al., 2008], that is applying said knowledge to related yet different problems [Perkins et al., 1992]. However, a replication of this study showed that the advantage of abstract representations disappeared when improving the concrete examples to make them more intuitive and less distracting [Trninic et al., 2020].

Beyond visual representations, manipulable representations have also been explored, and show great potential in mathematics education [Carbonneau et al., 2013]. In particular, the use of concrete manipulatives increases retention, problem solving, transfer, and justification scores over more abstract symbols [Carbonneau et al., 2013].

When grounding mathematical concepts in concreteness, the process used is also of importance. For example, simultaneous multiple representations can help students ground abstract content to more visual artifacts [Rau, 2017]. However, several issues might arise. First, the representation dilemma: as students have to conjointly learn the novel content and the novel representation, one has to ensure that the benefits of the novel representation exceed its cost [Rau, 2017]. Second, using representations that are related but spatially or temporally distant can result in a negative split attention effect [Sweller et al., 2011].

Another approach to grounding in concreteness is “concreteness fading”, an instructional design building sequentially from a concrete, specific, relatable example to the corresponding abstract, general representation [McNeil and Fyfe, 2012; Suh et al., 2020]. Concreteness fading was proven beneficial over using solely concrete examples or abstract representations and over progressing from abstract to concrete representations [Fyfe et al., 2014; Fyfe et al., 2015]. Traditionally, concreteness fading evolves from an enactive representation, to an iconic representation and concludes with a symbolic representation [Suh et al., 2020]. But other forms of concreteness could be explored as well, and, we believe, this field could also benefit from a clarification of the role of different aspects of concreteness in learning.

In this work, we explore the influence of different forms of concreteness on grounding, in the context of abstract mathematics. In particular, we focus on manipulated concreteness and embodied concreteness, described in the following section.

5.2.2 Embodied concreteness

In this section, we conceptualize embodiment as a form of concreteness via embodied cognition and embodied interaction theories. Specifically, we use the term embodiment to describe "embodied interaction supporting embodied cognition". Following the two pathways to learning described in Chapter 4, with this term we suggest that the interaction is designed so that the interaction meaning is congruent with the mathematical meaning.

How does embodiment fit in the concreteness space described above?

First, embodiment is focused on body movements and bodily manipulations of representations. This is true for all types of embodiment [Melcer and Isbister, 2016] and stronger for higher degrees of embodiment as they require higher sensorimotor engagement [Johnson-Glenberg and Megowan-Romanowicz, 2017]. For this reason, embodiment involves the sense of proprioception and, therefore, an embodied representation is concrete because it can be *sensed*. Embodiment can also relate to other senses. For example, eye-tracking studies revealed that gaze plays an important role in embodied learning [Abrahamson et al., 2015]. Moreover, certain embodied activities can include sound feedback [Antle et al., 2008] or haptic feedback [Müller et al., 2023].

Second, embodiment activities often target more specific elements and representations. For example, in Chapter 4, the levels of our game focused on specific functions. However, this is not a form of concreteness that is necessary for embodiment. For example, embodiment has also been explored for more general mathematical expressions [Sansonetti et al., 2021].

Third, proponents of embodied cognition also argue that cognition is situated: that is, the construction of knowledge happens through interaction with a temporal and physical environment [Roth and Jornet, 2013]. This aligns with Dourish's definition of embodied interaction, focused on the social and physical context of the interaction [Dourish, 2004]. In this chapter, we explore the context of the interaction. From this perspective, we argue that embodiment can be considered concrete when its context is designed to be relatable for the learner.

Hereinafter, we define “manipulated concreteness” as a form on concreteness that involves manipulation, but does not include a relatable context. In contrast, we define “embodied concreteness” as a form of concreteness that involves a high degree of embodiment, in a situated and relatable context.

5.3 Design

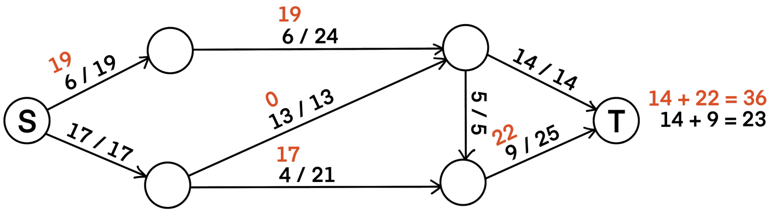


Figure 5.6: *Example of a valid flow graph, as well as suggested modifications (in red, above) to maximize the flow value of the graph.*

We started by designing and implementing an activity to ground graph theory in embodied concreteness. In this project, we focus on the max-flow problem [Sambol, 2015; Ford Jr. and Fulkerson, 1956], where, given a graph, the student has to maximize the amount of flow traveling from the Source (S) to the Sink (T), while respecting the maximum capacity of the edges, and the fact that vertices cannot store units (Figure 5.6).

In this section, we describe how we designed, implemented, and validated our embodied concreteness activity.

5.3.1 Concrete graph representation

Graphs, specifically flow networks, can be represented in various ways, such as symbolically and geometrically.

> **Definition (Flow Networks).** Given a direct graph $G(V, E)$, vertices $S, T \in V$ (where $S \neq T$), and a capacity function $c : E \to \mathbb{R}^+ \cup \{0\}$, the tuple (G, S, T, c) is called a (flow) network.

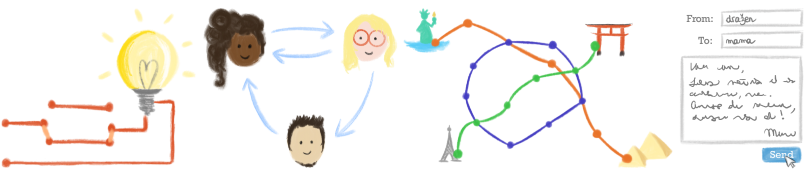
Graph problems can also be presented in a relatable manner, for example as electrical circuits, social networks, transportation networks, or internet networks (Figure 5.7). For our project, we focused on embodied concreteness, and therefore designed an embodied, sensed, situated, and relatable graph representation. To do so, we relied on embodied schemata from conceptual metaphor theory [Lakoff and Johnson, 2008]. According to Lakoff and Johnson, "the essence of a metaphor is understanding and experiencing one kind of thing in terms of another". In addition, embodied schemata are "recurrent patterns of bodily experience". Specifically, we looked for bodily experiences that could be used as metaphors for flows in graphs.

To our knowledge, embodied schemata have not yet been explored in the context of graph theory. However, in the case of electricity and electrical networks, two main schemata are used: WATER-FLOW and MOVING-CROWD [Gentner and Gentner, 1983]. Reusing these schemata in the context of graph theory is particularly relevant as flow networks are often used to solve electrical networks problems [Chen, 1997; Atkins et al., 2009; Dwivedi et al., 2010].

We used the WATER-FLOW schema as it is the most commonly used and therefore most relatable (Figure 5.8, left). In our activity, a graph is represented as a pipe network (edges) between water towers (vertices), and a simple simulation of the water flow through the pipes is displayed in real time. The goal for the student was to increase or maximize the amount of water flowing from the lake (source) to the city (sink). Upon success, a fountain placed at the entrance of the city starts pouring water.

5.3.2 Embodied interaction with a graph

As to attain a high degree of embodiment, we implemented the activity in Virtual Reality (VR) [Johnson-Glenberg and Megowan-Romanowicz, 2017], using hand tracking over controllers. Although interaction with graphs in



(a) Electrical circuit **(b)** Social network **(c)** Transportation network **(d)** Internet network

Figure 5.7: *Relatable examples of graphs.*

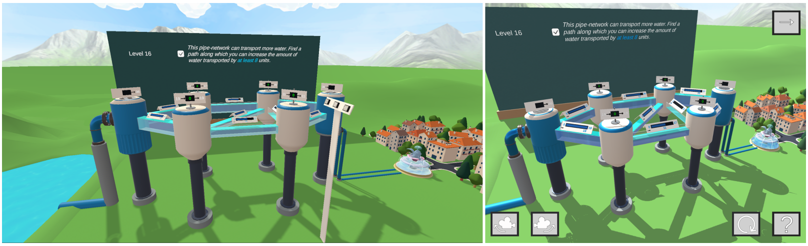


Figure 5.8: *Initial prototype of the embodied graph theory activity in Virtual Reality (Left), as well as the Tablet implementation used as a control for the usability evaluation (Right). In the Virtual Reality condition, the water flow in an edge is manipulated by holding the bottom of the pipe with one hand, and moving the other hand up or down to indicate the desired water level. In the tablet condition, the level is adjusted by touching the bottom of the pipe with one finger, and adjusting the level by moving another finger up and down.*

VR has already been explored, previous work focused more on data visualization and manipulation, and thus did not fit out project [Huang et al., 2017; Drogemuller et al., 2018]. Therefore, we designed our own system.

To design the embodied interaction with the edges of the graph, we looked at two approaches: direct-embodied and enacted [Melcer and Isbister, 2016; Ottmar et al., 2019]. The type of embodiment influences the mathematical concept emphasized, and impacts learning outcomes and persistence [Chatain et al., 2022]. As our activity focuses on the value of the flow in the edge, rather than its variation, we selected a direct-embodied approach.

During the development of the interaction, we evaluated various input mechanisms on a small group ($n = 6$) of participants who did not have a lot of experience with VR systems, in an informal setting. We gave the participants a short explanation on the input mechanism at the beginning and let them solve problems on their own. They gave verbal feedback on their experience, while we monitored their in-game activity. We wrote down the problems the participants faced during their time in the activity. After the use of the activity the participants explained their issues, thoughts and ideas in their own words regarding both the activity itself and the input technique. For the first participant, we used a one-handed input approach, setting the water level. Due to the imprecision when locking the water level, for further participants we introduced a two handed variant separating the adjustment and the locking movements. The locking was deactivated by removing the second hand from the pipe. We found that the participants were still struggling as they removed both hands at the same time. As a result, the interaction goes as follows: with one hand, the learner grabs the bottom of the edge with an open-close gesture, and by moving the other hand vertically, they can adjust the amount of water flowing in the edge. This two-handed interaction has two main advantages: it strengthens the embodiment as the capacity of the edge is directly congruent to the distance between the two hands, and it improves usability as the learner can release the lower hand to precisely set the level of the edge.

To improve usability and embodiment, we also adjusted the height of the entire graph to fit the height of the learner.

5.3.3 Pedagogical pattern

In our work, we focus on grounding, and therefore we needed to reconnect our activity with more formal forms of instruction [Chatain et al., 2022]. To do so, we decided to follow a Problem Solving followed by Instruction (PS-I) pedagogical pattern, where, as opposed to the more wide-spread I-PS pattern, the students solve exercises about the topic before receiving instruction [Loibl et al., 2017]. This pattern was proven effective for mathematics learning and relies on three mechanisms: activation of prior knowledge, awareness of knowledge gap, and recognition of deep features [Loibl et al., 2017; Sinha and Kapur, 2021]. In our activity, we activate concrete prior knowledge by offering a relatable experience to the students. Moreover, we designed our levels to increase knowledge gap awareness and identification of deep features. Indeed, each level either increases complexity to encourage search for more general solutions, or highlights new problem features such as the counter-intuitive need to decrease flow on certain edges to increase output flow. The Instruction phase of the PS-I pattern was handled differently for each study and is described in their respective sections. We selected a PS-I pedagogical pattern over a concreteness fading one as we were interested in the effect of different forms of concreteness on grounding and learning. Choosing to focus on concreteness fading would imply defining a different sequential pattern for each condition and shift the focus away from our research question.

5.3.4 Design Validation

We validated our design with a user study focused on usability. To do so, we designed a control condition on a tablet (Figure 5.8, right). In this condition, the representation is the same, but the embodiment is of a lower degree as immersion and sensorimotor engagement are reduced [Johnson-Glenberg and Megowan-Romanowicz, 2017]. For the tablet prototype, we replaced the two-hands interaction by a two-fingers interaction where one presses an edge with one finger, and adjusts its flow quantity with another finger. To mimic the navigation of the VR condition, we added two buttons to rotate the camera around the pipe network.

Demographics

We recruited $n = 26$ participants (6 identifying as female, 20 as male), from Zurich, Switzerland ($n = 9$), and Budapest, Hungary ($n = 17$). Participants were, in average, $M = 27.26$ years old ($SD = 8.17$) and were assigned to the tablet ($n = 13$) and VR ($n = 13$) conditions randomly. One participant in the tablet condition was removed from the analysis as she was an outlier in terms of time spent in the activity.

Protocol

We tested our prototypes within a PS-I pedagogical pattern. First, participants completed a general questionnaire including demographics questions, followed by a learning pre-test, and a Simulator Sickness Questionnaire (SSQ) [Kennedy et al., 1993]. Then, as a Problem Solving phase, the participants solved the graph theory problems with either the tablet prototype or the VR prototype. Afterwards, as an Instruction phase, the participants watched a short video on the Ford-Fulkerson algorithm [Sambol, 2015; Ford Jr. and Fulkerson, 1956]. Finally, participants completed a System Usability Scale (SUS) questionnaire [Brooke and others, 1996], a SSQ, as well as a learning post-test comprised of recall and transfer questions with different representations. This study was approved by the ETH Ethics Commission as proposal EK 2022-N-64.

Results

Aligned with previous work [Chatain et al., 2022], we found no significant differences in usability between the tablet and the VR prototypes ($p = 0.14$, $t(24.0) = 1.52$). The tablet prototype received a SUS score of 86.54 ($SD = 6.89$), qualified as “Excellent” [Bangor et al., 2009], while the VR prototype received a score of 81.35 ($SD = 10.19$), qualified as between “Good” and “Excellent”.

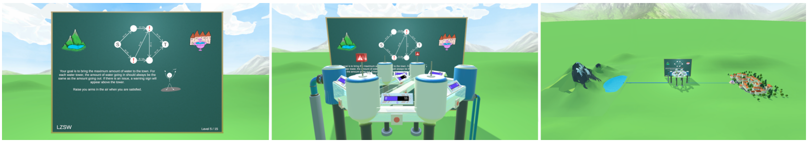


Figure 5.9: *Final prototype of the embodied graph theory activity in Virtual Reality. In this prototype, the water flow in an edge is manipulated by pressing the button at the bottom of the pipe with one hand, and moving the other hand up or down to indicate the desired water level.*

We did not find significant differences in SSQ scores either ($p = 0.33$, $t(24.0) = -0.97$). In particular, the tablet prototype can be categorized as generating "negligeable" symptoms, while the VR condition generates "minimal" symptoms [Stanney et al., 1997]. As our VR activity does not include fast-paced changes, simulator sickness is reduced [Stoner et al., 2011].

The VR activity took more time that the tablet activity ($p = 0.06$, $t(24.0) = -2.00$). This is also congruent with previous research [Chatain et al., 2022], and is justified by the fact that VR participants perform wider movements and moved around the space more.

5.3.5 Design improvements

Based on our observations during the study, we made several adjustments to our prototype (Figure 5.9). To improve embodiment and acknowledge the diversity of learners' bodies [Spiel, 2021], we added a skin color selection panel. To make the learners more confident in their movements, we added a tutorial where they can explore the virtual space [Chatain et al., 2022]. To improve usability, we made the direction of the edges clearer. As several participants reported struggling getting an overview of the problem, we added a depiction of the graph on the black board. We also added a button on the edges to make the interaction technique clearer and strengthen the embodiment (Figure 5.9, center). Finally, following the idea of "experiencing the body as play", we designed for a sense of embodied achievement at the end of each level by having the users adopt a "winning position", that is raising both arms in the air, to launch the next level (*Leib*) [Mueller et al., 2018].

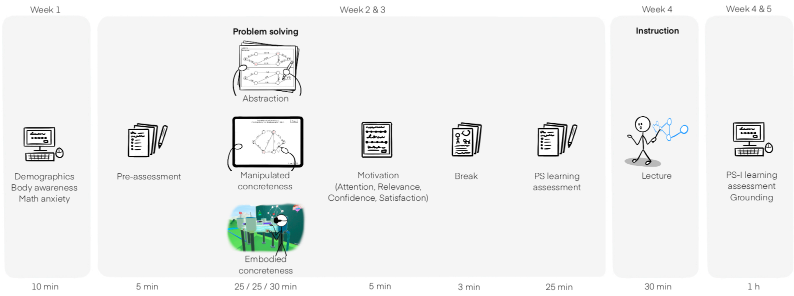


Figure 5.10: *Overview of the user study protocol.*

5.4 Comparison of Concretenesses

5.4.1 Research Questions

After validating our design, we used our activity to address the following research questions:

RQ1 What is the impact of concreteness on grounding?

RQ2 What is the impact of concreteness on learning outcomes?

In this work, we want to evaluate the impact of different forms of concreteness on grounding and learning. To do so, we focus on concreteness as a standalone Problem-Solving intervention in a PS-I pedagogical pattern. For this study, we designed three experimental conditions (Figure 5.1). In the abstraction condition (ABST), the students solve the exercises on paper, with a geometrical graph representation. This condition is the most abstract as it is not manipulated, embodied, situated, nor relatable for graph theory novices. We use this condition as a baseline to define the two concreteness conditions. In the manipulated concreteness condition (MNPL), we add feedback and interactivity: In this condition, the students solve the same exercises on a tablet, where they can interact with the graphs' geometrical representation. This condition is concrete as it is manipulated and embodied at a low degree. However, it is still not situated nor relatable. In the last condition, embodied concreteness (EMBD), the students now solve the exercises using our embodied graph theory activity, in a highly embodied, situated, and relatable manner.

5.4.2 Demographics

We recruited $n = 89$ (33 female, 54 male, 0 other, 2 undisclosed) volunteer bachelor students, aged $M = 20.6$ years old ($SD = 2.00$), from a Data Structures and Algorithms course at the mathematics department of ETH Zurich. The participation was rewarded by gaining access to a bonus exercise awarding extra points to the final exam. Participants were randomly assigned to each of the conditions ($n_{ABST} = 30$, $n_{MNPL} = 29$, $n_{EMBD} = 30$). The intervention included a pre-assessment questionnaire to evaluate previous knowledge on graph theory and the specific max-flow problem. This assessment was designed by the authors and included items such as "Have you learned graph theory previously?" with answers such as "I have, in a formal environment. (e.g. secondary school or university)", "I have, only informally. (e.g. self-study)", "No, I have not.". The assessment also included a max-flow problem to solve. Only 3 participants reported having learned about problems involving graphs before, and only one of them managed to solve the max-flow problem successfully. This participant was excluded from the analysis and is not included in the $n = 89$ sample size.

5.4.3 Protocol

We used a between-participants design to avoid learning effects across conditions. The study was composed of five steps: pre-intervention, intervention (Problem Solving phase), lecture (Instruction phase), exercises and post-intervention questionnaire (Figure 5.10).

During the pre-intervention, participants completed a questionnaire at home, including general demographics questions, a body awareness questionnaire [Shields et al., 1989], and a math anxiety questionnaire [Hopko et al., 2003].

The Problem-Solving intervention was conducted in our lab, in a separate room. For the EMBD condition, we prepared a VR space of 4m * 4.5m to accommodate all the levels of the activity. During the intervention, the participants signed a consent form and completed a pre-assessment asking about their knowledge of Graph Theory. They then solved max-flow problems in one of the three conditions: abstraction (ABST), manipulated concreteness (MNPL), or embodied concreteness (EMBD). Participants had 25 minutes to solve the problems, except EMBD participants who had 30 minutes in order to account for the calibration steps. For the MNPL and EMBD conditions, we logged the actions of the user. Then, participants filled in an Instructional Materials Motivation Survey following the ARCS model: Attention,

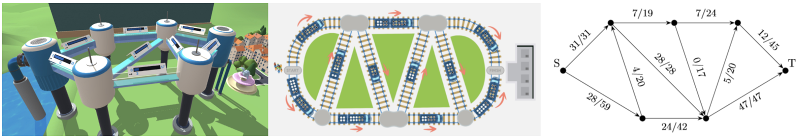


Figure 5.11: *Post-test representations: concrete WATER-FLOW embodied schema, concrete MOVING-CROWD embodied schema, abstract.*

Relevance, Confidence, and Satisfaction [Keller, 2010; Loorbach et al., 2015; Lab, 2010]. In order to alleviate fatigue effects, participants then took a three minutes break where they could read some selected comics. Finally, participants solved a 25 minutes learning assessment, evaluating the effect of the problem solving phase, and focusing on isomorphic problem-solving with different representations (Figure 5.11): concrete based on the WATER-FLOW embodied schema (similar to EMBD condition), concrete based on the MOVING-CROWD embodied schema (the graphs are represented as a train network with a flow of passengers), and abstract (similar to ABST and MNPL conditions) [Gentner and Gentner, 1983]. The questions were presented in a randomized order to alleviate effects undesirable within learning assessment, such as concreteness fading [Bruner and others, 1966; Fyfe et al., 2014].

About a week after the intervention, as an Instruction phase, participants followed a lecture on graph theory and max-flow problems, by their regular instructor, at their regular schedule.

A few days later, participants solved exercises on maximum flows on their usual exercise platform, and completed a questionnaire about the relevance and usefulness of the intervention for the lecture and the exercises.

This study was approved by the ETH Ethics Commission as proposal EK 2022-N-40.

5.4.4 Results

In the following sections, we performed one-way ANOVAs with the following contrasts: abstraction opposed to concreteness, and manipulated concreteness opposed to embodied concreteness. We checked for the assumption of normality with a Shapiro-Wilk Normality Test [Royston, 1982], and we checked for the assumption of homoscedasticity using a Breusch-Pagan Test [Breusch and Pagan, 1979]. If the assumptions were met, we used a regular ANOVA [Chambers et al., 2017] with Bonferroni post-hoc comparisons, otherwise we used a robust ANOVA [Wilcox, 2011] with Linear Constraints post-hoc comparisons.

To evaluate learning outcomes, we performed a Bayesian analysis to make sense of non-significant statistical tests [Faulkenberry et al., 2020; Jeffreys, 1961; Dienes, 2014; van Doorn et al., 2021]. To do so, we first performed a Bayesian two-sided analysis of variance, followed by post-hoc tests when necessary. Specifically, we considered equal prior odds and compared a null model (M_0) to a model considering the main effect of condition only (M_1). In the following, we use BF_{01} to describe the ratio $\frac{P(M_0)}{P(M_1)}$.

Grounding

To address RQ1, we first looked at the four components of the ARCS model [Keller, 2010]. Attention refers to how captivating and interesting the content is for the learners. Relevance refers to how valuable and connected to the real world the experience is. Confidence refers to how much the activity helped the learners feel in control of their success and likely to succeed. Satisfaction refers to how good the learners feel about their accomplishments and continuing to learn.

We found significant effects of condition on all four components (Figure 5.12): Attention ($F(2, 34.64) = 16.06$, $p < 0.001$), Relevance ($F(2, 86) = 21.72$, $p < 0.001$), Confidence ($F(2, 34.28) = 4.86$, $p = 0.014$), and Satisfaction ($F(2, 33.54) = 5.46$, $p = 0.009$). The effect sizes were large for Attention ($\xi = 0.64$) and Relevance ($\eta = 0.58$), and medium for Confidence ($\xi = 0.40$) and Satisfaction ($\xi = 0.45$).

Regarding Attention, there was a significant difference between ABST and MNPL ($p = 0.001$), as well as between ABST and EMBD ($p < 0.001$), but not between MNPL and EMBD ($p = 0.15$). Similarly, regarding Confidence, there was a significant difference between ABST and MNPL ($p = 0.034$), as well as between ABST and EMBD ($p = 0.028$), but not between MNPL and EMBD ($p = 0.81$). Regarding Satisfaction, we found a close to significant difference between ABST and MNPL ($p = 0.089$), and significant difference between ABST and EMBD ($p = 0.007$), but there was no significant difference between MNPL and EMBD ($p = 0.22$). In contrast, for the Relevance component, we found no significant difference between ABST and MNPL ($p = 0.41$), but we found a significant difference between ABST and EMBD ($p < 0.001$) as well as between MNPL and EMBD ($p < 0.001$).

We also looked at the results of the grounding questionnaire (Figure 5.13). This questionnaire included 5 points Likert scale items such as “Did the ac-

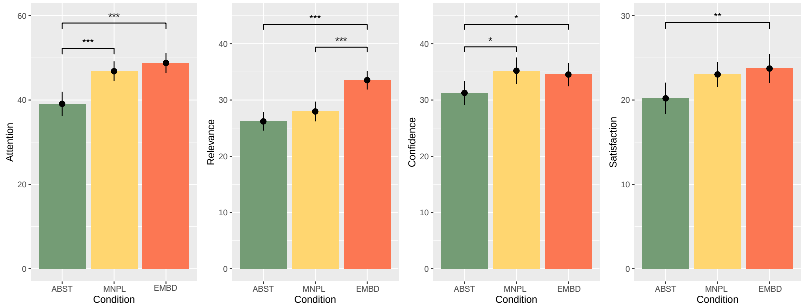


Figure 5.12: *Bar plot representation of the ARCS model per condition (abstraction, manipulated concreteness, embodied concreteness), with adjusted p-values (*p < 0.05, **p < 0.01, ***p < 0.001), and 95% confidence intervals.*

tivity make you excited about joining the lecture?" or "Do you feel that the activity prepared you for the lecture?". Similar items about solving the final exercises were included.

We found a significant effect of condition on the grounding items related to the lecture ($F(2, 28.53) = 3.87$, $p = 0.033$), with a medium effect size ($\xi = 0.38$). The post-hoc analysis only revealed significant difference between the ABST and the EMBD conditions ($p = 0.030$), while no significant difference was revealed between the ABST and MNPL conditions ($p = 0.23$), nor between the MNPL and EMBD conditions ($p = 0.23$).

We did not find a significant effect for the items related to the exercises ($F(2, 75) = 0.60$, $p = 0.55$), but, although many participants answered the questionnaire, too few participants actually solved the exercises. Therefore, we refrain from drawing any conclusions regarding this aspect.

In conclusion, regarding RQ1, both forms of concreteness significantly improved Attention, Confidence, and Satisfaction for the learners. However, only embodied concreteness improved perceived Relevance. Moreover, only embodied concreteness improved perceived grounding after the lecture. This is particularly important as students often believe that mathematics has nothing to do with the real world [Schoenfeld, 2016]. Our results show that this issue can be alleviated using embodied concreteness.

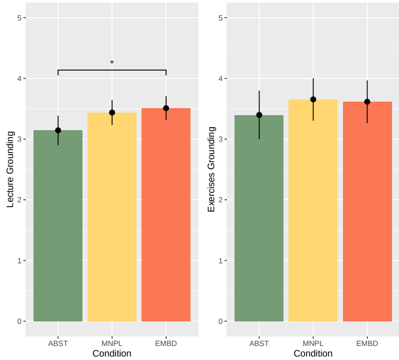


Figure 5.13: *Bar plot representation of the perceived grounding after the lecture and the exercises, with adjusted p-values (*$p < 0.05$), and 95% confidence intervals.*

Learning Outcomes

To address RQ2, we focused on learning outcomes. This is particularly important as previous research shows that the use of concrete examples may reduce the transfer capabilities of the students [Kaminski et al., 2008], and that, according to cognitive load theory [Sweller, 1994], high-immersive gaming environments such as VR might impair learning outcomes [Kuipers et al., 2017]. As too few participants completed the PS-I learning assessments, we did not include this test in our analysis.

Following the aforementioned procedure, we performed ANOVAs with respective subsequent comparisons on the different learning assessments (Figures 5.11, 5.14). We found no significant effect of condition in the following learning assessments: WATER-FLOW representation ($F(2, 34.65) = 0.10$, $p = 0.90$) and MOVING-CROWD representation ($F(2, 34.06) = 0.48$, $p = 0.62$). The only significant effect was on the abstract representation ($F(2, 34.3) = 3.40$, $p = 0.045$), and it was of medium size ($\xi = 0.33$).

As absence of evidence is not evidence of absence, we performed a Bayesian analysis to estimate whether there is indeed no performance difference between the groups on the WATER-FLOW and MOVING-CROWD learning assessments. Our analysis revealed moderate evidence for the null hypothesis of no effect on the WATER-FLOW assessment ($BF_{01} = 9.706$) as well as the MOVING-CROWD assessment ($BF_{01} = 6.834$).

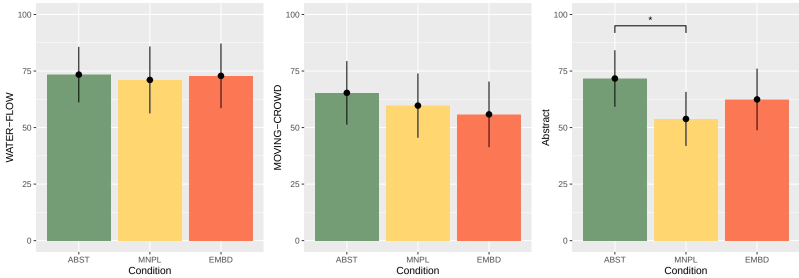


Figure 5.14: *Bar plot representation of the learning outcomes for different representations, with adjusted p-values (*p < 0.05), and 95% confidence intervals.*

On the abstract representation exercises, students in the ABST condition ($M = 71.7$, $SD = 33.5$) outperformed students in the MNPL condition ($M = 53.81$, $SD = 31.5$) significantly ($p = 0.04$). However, there was no significant difference between the ABST condition and the EMBD condition ($M = 62.45$, $SD = 36.5$, $p = 0.34$), with anecdotal evidence for the null hypothesis ($BF_{01} = 1.982$). The post-hoc tests revealed anecdotal evidence of no effect between ABST and EMBD ($BF_{01} = 2.457$), as well as between MNPL and EMBD ($BF_{01} = 2.548$). However, we observed anecdotal evidence for the alternative hypothesis between ABST and MNPL ($BF_{01} = 0.603$).

In conclusion, there was no effect of the condition on learning outcomes with concrete representations. However, students learning with manipulated concreteness performed worse on abstract tasks, while this difference was not significant for students learning with embodied concreteness. This means that embodied concreteness did not impair the capabilities of the students to transfer to a different embodied schema. It seems that embodied concreteness also did not impair transfer to a more abstract representation, although the evidence is only anecdotal at this point. However, manipulated concreteness had a negative impact on transfer, possibly as students relied too much on feedback, preventing them from thinking deeper about the problem.

5.5 Discussion

In our work, we described several perspectives on concreteness and abstraction, and argued that there is a verbal dispute in the field on mathematics education. While many experts explore the role of concreteness in mathematics learning and teaching, the terms "concrete" and "abstract" are often underspecified, resulting in mixed results and ambiguity. We then argued that embodiment, that is embodied interaction for embodied meaning-making, can be explored as a form of concreteness.

To illustrate our discussion, we designed and implemented an embodied activity targeting the max-flow problem in graph theory. We then compared three approaches to problem-solving: abstraction, manipulated concreteness, and embodied concreteness. Our results demonstrate that different aspects of concreteness have different impacts on grounding and learning outcomes.

In the rest of the section, we discuss the mechanisms of learning with concreteness, the impact of our work, as well as its limitations, and provide suggestions for future work.

5.5.1 Mechanisms of learning with concreteness

Following on the outcomes of our user study, we discuss what mechanisms are responsible for these results, and in particular, which affective and cognitive mechanisms are activated by different kinds of concreteness. In the following, we only focus on the mechanisms involved in the problem solving phase, as looking at the entire PS-I pattern would be beyond the scope of our project, and has already been explored in previous work [Sinha and Kapur, 2021].

In our project, we explored two forms of concreteness. In the manipulated concreteness, students could manipulate a graph representation, with a low degree of embodiment. The system would give them limited feedback, for example prevent them from exceeding the capacity of an edge, or indicate when a node is invalid. In the embodied concreteness condition, the students could manipulate a situated and relatable representation of a graph, with a high degree of embodiment. The provided feedback included more information, as a water flow was also simulated along the pipes composing the graph.

As a result, different mechanisms should be considered for each of these conditions, summarized in Table 5.1. First, there are several mechanisms related to feedback only. Indeed, feedback supports error identification [Mory, 2013] and strategy acquisition [Fyfe et al., 2012]. Moreover, while the effect of low-information feedback is usually low, high-information feedback has a stronger impact as it supports error understanding [Wisniewski et al., 2020].

Second, as explained in Chapter 2, from a representation-agnostic standpoint, embodiment involves three main mechanisms [Körner et al., 2015]. Direct state induction is a mechanism of embodiment relying on the fact that certain bodily states impact the feelings of the learner independently of any cognitive mechanism. We support this mechanism in our embodied activity as we designed the experience from the feeling body (*Leib*) perspective [Mueller et al., 2018], for example by inducing a feeling of embodied achievement as the learners adopt a winning position to finish a level. In turn, modal priming is a mechanism through which sensorimotor states enable learners to access abstract concepts, for example via conceptual metaphors. In our project, this mechanism is activated through the WATER-FLOW embodied schema [Gentner and Gentner, 1983; Lakoff and Johnson, 2008]. Finally, sensorimotor simulation is a mechanism of embodiment through which congruent bodily states and actions ease subsequent mental simulations, in particular in the context of problem solving [Dijkstra and Post, 2015]. We reconnect this particular mechanism to the conceptualization of thinking as truncated action [Abrahamson and Lindgren, 2014]. In this new light, sensorimotor simulation describes how sensorimotor experiences can support further truncated actions, and therefore, thinking.

Table 5.1: *The mechanisms of learning with concreteness.*

<table>
<tr><th>Condition</th><th>Concreteness</th><th>Mechanisms</th></tr>
<tr><td>Abstraction</td><td></td><td></td></tr>
<tr><td rowspan="2">Manipulated concreteness</td><td>Low-information feedback</td><td>Error identification
Strategy acquisition</td></tr>
<tr><td>Embodiment (low degree)</td><td>Sensorimotor simulation (low degree)</td></tr>
<tr><td rowspan="2">Embodied concreteness</td><td>High-information feedback</td><td>Error identification
Error understanding
Strategy acquisition</td></tr>
<tr><td>Embodiment (high degree)</td><td>Direct state induction
Modal priming
Sensorimotor simulation (high degree)</td></tr>
</table>

In our study, we showed that both forms of concreteness increased Attention, Confidence, and Satisfaction similarly. Therefore, these results should be explained by the common mechanisms between the two conditions: error identification, strategy acquisition, and sensorimotor simulation.

In turn, only embodied concreteness increases perceived relevance and grounding. We believe that this is explained by the modal priming mechanism as it reconnects the content to the learners' personal experiences, which is an important aspect of relevance in learning [Sharma et al., 2022]. Moreover, relevance can be defined as a continuum of personal association, personal usefulness, and identification, and can trigger different mechanisms based on personal differences [Sharma et al., 2022; Priniski et al., 2018]. In future work, such mechanisms should be explored in more depth in order to provide a more detailed account of the mechanisms of embodied concreteness.

Finally, learning with manipulated concreteness reduced the learning outcomes on abstract representations. We believe that this is explained by the lack of error understanding mechanism in this condition. In particular, as the representation was familiar to the students, they felt confident about solving the problems, and therefore, we believe, were more prone to errors.

5.5.2 Interaction context in learning

We can also consider these results within our framework (Figure 2.10). In this context, both manipulated concreteness and embodied concreteness involve meaningful interaction, in the sense that the movement of the body is congruent with its effect on the virtual world. This supports the following path in our framework: `Context` $\rightarrow$ `Interaction` $\Rightarrow$ `Meaning making` $\Rightarrow$ `Learning`. However, in the embodied concreteness condition, the context also gives meaning to the bodily action through modal priming, and activates the following path: `Context` $\rightarrow$ `Interaction` $\rightarrow$ `Bodily action` $\Rightarrow$ `Learning`. Therefore, when designing the context of the interaction, it is not sufficient to only consider embodied interaction and mechanisms of embodied cognition should also be considered.

5.5.3 Impact

With this work, we hope to impact the field of concreteness in mathematics in two ways.

First, we illustrated the need for a more rigorous definition of "abstraction" and "concreteness" in the field of mathematics education. In future work, we believe that a taxonomy of concretenesses should be defined, for example, building on a categorization framework of different representations along aspects of groundedness and idealization [Belenky and Schalk, 2014]. Moreover, we saw that concreteness can be defined as a property of the object only (concrete as specific), but also through the interaction of a learner with the object (concrete as tangible), or the mental model the learner has of the object (concrete as relatable). This aspect could be deepened if reconnected to the theory of affordances, building on the similar distinction between the Gibsonian and the Normanian perspectives [Gibson, 2014; Norman, 2013]. Such tool should then be used to support a meta-analysis of previous work on concreteness and mathematics education, and identify which aspects of concreteness, and related affective and cognitive learning mechanisms, specifically impact learning.

Furthermore, investigating abstraction is at least as important, as the link between concreteness and abstraction is not necessarily dual, and similar verbal dispute exists for abstraction. For example, although abstraction is often conceived as a Platonic overarching, perfect ideal or truth, more recent work on abstraction offers an alternative grounded in mathematics history. For example, according to Wagner, mathematical abstraction can be defined as [Wagner, 2019]:

> [Abstraction is] the practice of incomplete, underdetermined, intermittent and open-ended translations between systems of presentations.

Moreover, vagueness, another word often associated with abstraction, can actually be formalized within the mathematical framework as vague or fuzzy mathematics [Syropoulos and Tatsiou, 2021]. Exploring different forms of abstraction would be particularly impactful within the field of concreteness fading [McNeil and Fyfe, 2012].

In addition, "concreteness fading" focuses on a vertical paradigm: that is showing the *way* to abstraction, starting from a concrete example. However, as we highlight different meanings of concreteness and abstraction, this vertical paradigm might not be best suited to explore the role of concreteness in math education. In particular, and in alignment with Wagner's definition of abstraction [Wagner, 2019], considering a horizontal paradigm–that is, providing diverse relevant concrete examples to elicit a *reason* for abstraction–might be more relevant. This distinction is important for the learner, as a vertical paradigm appears as a mere translation between concrete and abstract representations, whereas a horizontal paradigm highlights the power of abstraction: its reusability. Artigue identified this issue in mathematics education at the University level [Artigue, 2009]:

> The results tend to favor a 'vertical' and hierarchical vision of mathematical learning and consequently to mask the importance of what one might like to describe in the 'horizontal' dimension.

Second, we showed that, although different kinds of concreteness can improve learners' attention, confidence, and satisfaction, embodied concreteness is a uniquely powerful tool for grounding mathematics as it increases perceived relevance while not impairing learning outcomes and transfer to more abstract representations. With embodied concreteness, learners can connect abstract concepts to real world experiences, thus challenging their unproductive beliefs about mathematics [Schoenfeld, 2016]. In future work, other comparisons should be explored. For example, comparing relatable but disembodied opposed to relatable and embodied would help isolate the effect of the modal priming mechanism in embodiment.

5.5.4 Limitations and Future Work

The main limitation of our work is the use or different technologies for the different conditions (paper, tablet, and VR). This was a conscious decision as we found important to offer a fair comparison by selecting the most appropriate technology for the activities we wanted to design. Indeed, offering non-manipulable graphs in VR, a technology heavily focused on bodily manipulations, would create unnecessary fatigue and confusion for the users. Starting from the paper baseline, we believe tablet is the best technological solution to add interaction and feedback to the activity. Similarly, VR is better suited to add a high degree of embodiment to the activity. However, our solution is not perfect: different technologies come with different effects, that are non-negligible, such as novelty effect in the case of VR [Huang, 2020]. We believe this issue is mitigated as our usability study shows that there is no significant difference between our activity in VR and on tablet. However, to complement this work, further studies should investigate the role of technology in these results. Moreover, our study would have been stronger by adding a manipulable condition on tablet using the relatable representation used in VR in order to better isolate the role of the modal priming mechanism. Unfortunately, our sample size did not allow for an extra condition. We believe it would be interesting to conduct this study in the future.

Another difference between our condition is the use of an overview graph in the embodied concreteness condition. This overview was added as our preliminary study revealed a perspective issue in this condition: on the tablet, the learner could see all the graph, while in VR, the learner had to navigate the space to build a global understanding of the problem. Another solution would have been to reduce the size of the network altogether. While we explored this solution through informal testing, we noticed that the lack of precision due to the hand tracking made the experience frustrating to the users. This might impact the learning outcomes, either by generating a negative split attention effect [Sweller et al., 2011], or by inducing a positive indexing effect [Hornecker, 2016].

To address these limitations, the design of the second study could have been improved by adding another condition using the relatable pipe system representation, but on tablet, similar to the tablet condition of our usability study. This way, we would have been able to isolate the effect of manipulation from the effect of representation. However, in light of the context of our study, we only had access to a limited number of participants. Based on our power analysis, we could not afford reducing the number of participants per condition by adding another condition. We selected these specific conditions for our learning study for two reasons: First, we wanted to evaluate the potential of embodied concreteness for grounding, second, we wanted to isolate the effect of interaction and feedback as compared to a paper condition. However, to complete this contribution, future work should explore the role of representation decoupled from interaction.

Several other aspects could be improved in future work. First, the Problem Solving part of our intervention was conducted in our lab, which is not an ecologically valid environment. Second, in our study we only looked into short-term learning outcomes. Measuring learning outcomes over several months might reveal differences, for example, the effect of grounding on long-term learning outcomes. Moreover, we believe that including embodied assessments in the study design might reveal interesting insights. Indeed, learners are usually able to express understanding through gestures before they can articulate it with speech [Nathan et al., 2022; Roth, 2001; Church and Goldin-Meadow, 1986], and embodied assessments would capture this effect. Finally, the assessments were limited in time, and in English. This could have biased some of the results, in particular for slower students and non-native English speakers.

Another concern is the diversity of our sample. For example, only few women participated (23% in the first study, 37% in the second one). This is mostly due to our recruitment. For example, for the second study, we wanted to focus on participants from our target group, we recruited students' from the Mathematics department, already suffering from a gender diversity issue (23% women). Moreover, we only tested our approach with students from a mathematics Bachelor program. However, mathematics is a field of importance, even to those who do not wish to become mathematicians. Future work should focus on performing a similar study with a population less intrinsically motivated by the field. Another important concern to raise is that our sample only included able-bodied participants. However, a wider diversity of bodies should be included in embodiment research [Spiel, 2021].

Finally, previous work identified the need to reconnect concreteness with more abstract representations [Fyfe et al., 2014; Fyfe et al., 2015; Chatain et al., 2022]. In our studies, we did so using videos or lectures, but building this connection directly in the embodied activity might facilitate transfer. Moreover, we focused on concreteness in a standalone intervention, and we did not explore the impact of these different forms of concreteness within a sequential pattern. Future work should investigate embodied concreteness fading, for example using tools for embodied input of mathematical expressions [Sansonetti et al., 2021].

5.6 Conclusion

In this chapter, we explained that students often struggle to grasp mathematics as it heavily relies on abstract symbols and formalisms that only gain meaning when grounded in concreteness. We then highlighted a verbal dispute in the field of concreteness in mathematics education, as the word "concrete" is not always used with the same meaning. In particular, we presented different kinds of concreteness and described embodiment as a powerful form of concreteness as embodied experiences support meaning-making through interaction with relatable objects and environments.

To support our argument, we created an activity to solve graph theory problems in an embodied manner, and validated our design with a first user study, revealing its high usability. We then used our activity to demonstrate the effect of different kinds of concreteness. Through this second user study, we compared three conditions: abstraction, manipulated concreteness, and embodied concreteness. Our results show that both forms of concreteness can increase learners' attention, confidence, and satisfaction. However, only embodied concreteness increases perceived relevance and supports grounding. Moreover, unlike manipulated concreteness, embodied concreteness did not negatively impact performance on abstract representations.

With this work, we contribute to the field of mathematics education in two ways. First, we illustrate the importance of rigorously distinguishing different kinds of concreteness. Second, we provide empirical evidence supporting embodied concreteness as a powerful tool to ground abstract mathematics. Coming back to the research question addressed in this thesis, the work presented in this chapter shows that designing a relatable context for the embodied interaction supports grounding and learning of abstract mathematical concepts.

For further reference, a version of this chapter has been published independently [Chatain et al., 2023c].

Designing to account for individual differences

Although mathematics is often considered disembodied, research in embodied cognition highlights the importance of learners' bodies in learning. However, moving does not necessarily translate to learning, and most empirical work fails to demonstrate significant learning outcomes of embodied learning activities over their counterpart. This chapter aims to shed light on bodily actions performed by learners in embodied activities and offer design recommendations for future work. To cover the design space, we consider both directed and spontaneous embodiment. Our results highlight the need to expand embodied interaction beyond position and movement, consider embodied metaphors and embodied concreteness, allow for coarse gesturing and repetitions, support and evaluate sense-making anchors, and integrate embodied assessments. We believe this exploratory work will impact the field of embodiment by opening new avenues of research and offering adequate solutions.

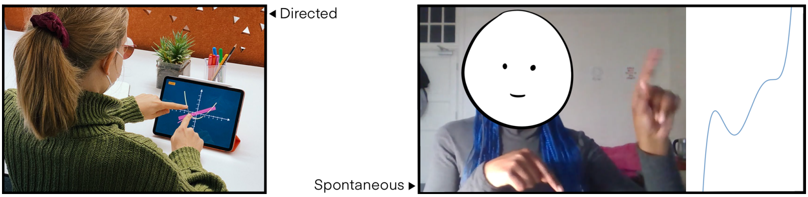


Figure 6.1: *Directed bodily actions are performed as explicitly asked by a task. Spontaneous bodily actions are performed spontaneously while solving a task.*

6.1 Introduction

In the previous chapters, we explored the role of avatar, interaction, and context design in embodied activities. However, although our work showed several advantages of embodiment for learning, such as grounding in concreteness, our empirical studies fail to identify the advantage of embodied activities in terms of learning outcomes. Specifically, we found no significant differences between embodied conditions and more abstract or less embodied conditions.

More generally, although the theoretical background for embodiment is growing and the technological capabilities are ready to support it, there are only a few quantitative evaluations of learning outcomes in embodied math activities, and the rare ones fail to convince [Ale et al., 2022; Chatain et al., 2022; Chatain et al., 2023c].

In this chapter, we pursue the investigation and understanding of the design space tied to embodied learning activities. Instead of focusing on interaction, we focus on the learners and provide an in-depth exploration of bodily actions performed as they make sense of mathematics. First, we consider *directed* bodily actions: actions that are performed as requested per the embodied learning activity. Specifically, we consider whether all learners perform the same actions or whether individual differences impact movement. Second, in the same fashion, we focus on bodily actions that learners *spontaneously* generate as they make sense of a concept. The corresponding desing space if presented on Figure 2.2 (right). In this exploratory chapter, we account for both the directed and the spontaneous perspectives and address the following question:

RQ How do learners move when making sense of mathematics and how can interaction design support such bodily actions?

Based on this exploration, we offer design recommendations accounting for the diversity of learners and detail avenues of research for future work.

6.2 Learners and Individual Differences

Embodied activities rely on sensorimotor simulation [Körner et al., 2015] and engagement [Johnson-Glenberg and Megowan-Romanowicz, 2017], and therefore their impact depends on how learners perceive and interact with space [Keehner and Fischer, 2012; Spiel, 2021]. Indeed, individual differences may influence the way people embody and manipulate mathematical concepts in embodied learning activities.

In this chapter, we address this aspect and consider the impact of individual differences on bodily actions in embodied learning activities. Specifically, our first study focuses on math anxiety, body awareness, and math ability. In this section, we describe the relevance of such factors to evaluate embodied activities.

6.2.1 Math anxiety

Math anxiety is "a feeling of panic, helplessness, paralysis and mental disorganization that arises when one is required to solve a mathematical problem or manage numbers" [Ashcraft et al., 1998; Chatain et al., 2022]. It develops as early as elementary school and persists or increases into adulthood [Dowker et al., 2016; Zhang et al., 2019]. Women, in particular, report more math anxiety than men [Barroso et al., 2021]. The effect of math anxiety is complex, it may lead to worse mathematical achievement, however, there are also high performing but math anxious individuals [Dowker et al., 2016; Zhang et al., 2019; Cipora et al., 2015; Rossi et al., 2022]. Math anxiety is a problem in and beyond classrooms. Therefore, teachers and researchers are committed to designing learning environments that help students reduce their math anxiety, have a positive learning experience, and improve their learning. Specific ways are recommended to teachers to make learning more meaningful, increase interaction, and reduce math anxiety; for example, the use of manipulatives, music, and games [Tate, 2008], or also the use of technologies such as Augmented or Virtual Reality [Salinas and Pulido, 2016; Finlayson, 2014]. Following the embodied cognition view and human development in general, movement is a promising factor in facilitating learning in mathematics. Movement, for example, improves spatial thinking and reasoning. Combined with math concepts, it might support learners' understanding of their bodies in space [Rosenfeld, 2017]. Furthermore, designing a learning environment in such a way that movement and mathematics are combined allows the learners to make meaningful connections between math to other areas of life and increases enjoyment and excitement about math [Kaufmann and Dehline, 2014]. This potentially leads to positive experiences with math learning and reduces math anxiety. Another study showed that movement could indeed help math-anxious learners [Isbister et al., 2012]. In this study, a mathematical game was played by learners who had to take in a power pose, standing with high raised hands, or a lower pose, sitting and using a pad to interact with the game. The power pose boosted confidence, while the playfulness was supposed to relax the students. Thus, a playful learning environment supporting bodily movements might be the way to go to decrease math anxiety while increasing interaction and engagement with math.

6.2.2 Body awareness

In this work, we also consider body awareness, that is, one's "attentiveness to normal body processes" [Shields et al., 1989]. We consider this an important factor in our endeavor as learners with different body awareness might rely on different cues when learning with embodied activities. For example, dancers, who usually have high body awareness, rely more on proprioceptive feedback than visual feedback [Jola et al., 2011]. Moreover, body awareness plays a major role when implementing embodied learning activities in Virtual Reality (VR). Indeed, in this context, learners manipulate digital content through an avatar. The difference between the learners' bodies and their digital representation is important and evaluated by the sense of embodiment [Kilteni et al., 2012]. The sense of embodiment relies on three aspects: The sense of self-location describes whether the learners feel as if their bodies are located in their digital counterparts; the sense of body ownership describes whether they feel like their digital bodies are their own; the sense of agency described whether they feel in control of their digital bodies. Body awareness plays a role in the sense of embodiment. In particular, there seems to be a negative correlation between body awareness and the sense of body ownership [Chatain et al., 2022].

6.2.3 Math ability

Another important factor to consider is math ability. Indeed, math ability plays an important role as learners with different math abilities move differently when making sense of or communicating mathematical concepts. In their study, Gerofsky et al. asked students and teachers to use bodily actions to describe a function's graph [Gerofsky, 2011]. They observed that high-achieving students and teachers used coarser movements, around the waist area, while hard-working students with lower grades performed precise movements at eye level. Another study on geometry proofs revealed that experts performed more representational gestures than non-expert, specifically dynamic representational gestures [Nathan et al., 2021]. Finally, recent work explored whether experts can be identified by their gesturing only and revealed that although experts produce fewer gestures than novices overall they produce more iconic gestures [Sriramulu et al., 2019].

The goal of this work is to gain insights on how learners move while making sense of mathematics, specifically derivatives. To do so, we performed two exploratory analyses. In our first study, we focus on directed embodiment and explore the impact of math anxiety, body awareness, and math ability on bodily actions in an intuition building activity. In our second study, we focus on spontaneous embodiment and identify key characteristics of gestures generated during an intuition probing task. We conclude by offering design recommendations and avenues for future research. Within our framework (Figure 2.10), the first study explores the following path: `Bodily action` → `Learner`, while the second study focuses on: `Learner` → `Bodily action`.

6.3 Directed Bodily Actions Analysis

First, we focused on directed bodily actions, that is bodily actions performed because the task explicitly requests them [Walkington et al., 2022; McNeill, 1992]. To do so, we collected more data using our embodied learning activity on derivatives and the same data collection protocol (Chapter 4 or [Chatain et al., 2022]). In this section, we offer a reminder of the activity design, including details important for this new analysis, and the data collection process. Please refer to the original chapter or paper for more details [Chatain et al., 2022]. In turn, we describe our analysis and results.

6.3.1 Embodied Derivatives

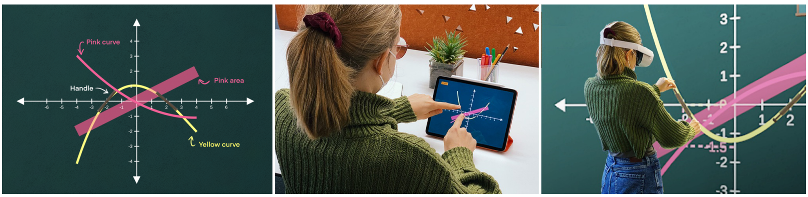


Figure 6.2: *Each level of the activity contains a yellow curve, representing the function, and a pink curve, representing the derivative. The goal is to manipulate the handles on the yellow curve to fit the pink curve into the pink target (Left). This activity was implemented on a tablet (Center) and in VR (Right).*

Activity This activity was designed to teach derivatives to high school students through an embodied game. In each level, a function curve in yellow and a corresponding derivative curve in pink are displayed. On the derivative curve, a target area is displayed, in pink, with lower opacity. The students are instructed to "explore the relationship between the yellow and the pink curve", and they can do so by solving puzzles where they manipulate the function curve to fit the derivative curve in the target area. On the function curve, several wooden handles can be manipulated to influence the shape of the function curve following a cubic spline model [Kruger, 2003]. At any point in time, the handle's slope corresponds to the local slope of the curve, and, therefore, the derivative (Figure 6.2).

The activity addressed several learning goals, labeled as "Core concepts" in Table 4.2. Moreover, the activity was implemented as part of a Problem Solving followed by Instruction (PS-I) pedagogical pattern. In this context, the activity served as a Problem-Solving phase and emphasized the three necessary mechanisms [Loibl et al., 2017]: prior knowledge activation by reusing curves that the students were familiar with, deep feature recognition by focusing the interaction on characteristic points of the function's curve, and knowledge gap awareness by including delayed feedback levels. Specifically, the activity contained two types of levels. The normal levels provided an immediate update of the function curve and the derivative curve upon manipulation. The delayed-feedback levels only updated the curves' shapes and positions upon the release of the handle. The different levels are summarized in Table 6.1.

Finally, the activity was implemented under two conditions (Figure 6.2). One condition, TAB, was implemented on a tablet and focused on a lower degree of embodiment [Johnson-Glenberg and Megowan-Romanowicz, 2017], with lower sensorimotor simulation and lower immersion. In contrast, a high degree condition, VR, was implemented in VR, with high sensorimotor simulation and high immersion. This condition was labeled DIR in the Chapter 4 but was renamed for clarity in this chapter.

Data Reusing our materials, we conducted a user study with $n = 149$ high school students. The protocol included two interventions. The first intervention included a prerequisite test, a demographics questionnaire, a math anxiety questionnaire [Hopko et al., 2003], a body awareness questionnaires [Shields et al., 1989], and a VR initiation focusing on hand tracking. The second intervention included a Simulator Sickness Questionnaire (SSQ) [Kennedy et al., 1993], the activity itself, a SSQ, a System Usability Scale (SUS) questionnaire [Brooke and others, 1996], a sense of embodiment ques-

Table 6.1: *Summary of the levels of the activity.*

Id	# Handles	Feedback	Primary goal	Notes
0	1	Normal		Tutorial, Linear function
1-2	1	Normal	UP	(idem)
3	1	Delayed	UP	(idem)
4-5	1	Normal	DOWN	(idem)
6	1	Delayed	DOWN	(idem)
7-8	1	Normal	FLAT	(idem)
9	1	Delayed	FLAT	(idem)
10	3	Normal	UP	Hyperbola, Handle at $x = -3$ is already correct
11	3	Delayed	UP	(idem)
12	3	Normal	DOWN	(idem)
13	3	Delayed	DOWN	(idem)
14-16	2	Normal	VAR	Parabola
17-18	2	Delayed	VAR	(idem)
19-20	3	Normal	SLOPE	Hyperbola, Handle at $x = -1$ is already correct
21	3	Delayed	SLOPE	(idem)

tionnaire [Roth and Latoschik, 2019], an agency questionnaire [Gonzalez-Franco and Peck, 2018], a short instruction video on derivatives, a break, and a post-test including questions from a Calculus Concept Inventory (CCI) [Epstein, 2007].

As some students missed the second intervention due to sickness or absence, our final sample size is $n = 130$. In average, the participants were $M = 17.10$ years old ($SD = 0.61$). 63 participants identified as female, 64 as male, 0 as other, and 3 unspecified. All of the participants were high school students, and the intervention was conducted a few weeks before the lecture on derivatives: this means that the students already studied graphs of functions, but did not study derivatives yet. The participants were split into two conditions: $n_{TAB} = 66$ participants were assigned the lower degree of embodiment condition, on tablet, and $n_{VR} = 64$ participants were assigned the higher degree of embodiment condition, in VR.

6.3.2 Research questions

We used this data to perform an exploratory analysis to understand the role of individual factors on how students interact with embodied sense-making activities, focusing on directed bodily actions for derivatives. From this analysis, we hope to gather an in-depth understanding of the behaviors to account for when designing embodied learning activities.

Specifically, we addressed the following research questions:

RQ1.1 How does math anxiety impact bodily actions?

RQ1.2 How does body awareness impact bodily actions?

RQ1.3 How does math ability impact bodily actions?

For clarity, this section focuses on how we analyzed the data, while our results are presented in the next section.

Individual factors

As defined in our research questions, we focused on math anxiety, body awareness, and math ability. These metrics were acquired one week before the intervention. Math anxiety was measured using the Abbreviated Math Anxiety Scale [Hopko et al., 2003]. Body awareness was measured using the dedicated body awareness questionnaire [Shields et al., 1989]. We used self-reported math grades as a proxy for math ability.

Preliminary embodied exploration

To perform an initial exploration of the data, we implemented visualizations of the students' actions for each condition. With our system, we could explore the embodied trajectories in space and time for all the students, both on tablet and in VR. We present static examples in Figure 6.3. This approach, recommended in embodied analysis and fused twins research [Grübel et al., 2022], helped us understand the kinds of movements the students performed and, therefore, define our metrics.

Behavior metrics

First, we considered which information is valuable for our behavior analysis. From the log information, we had access to the following:

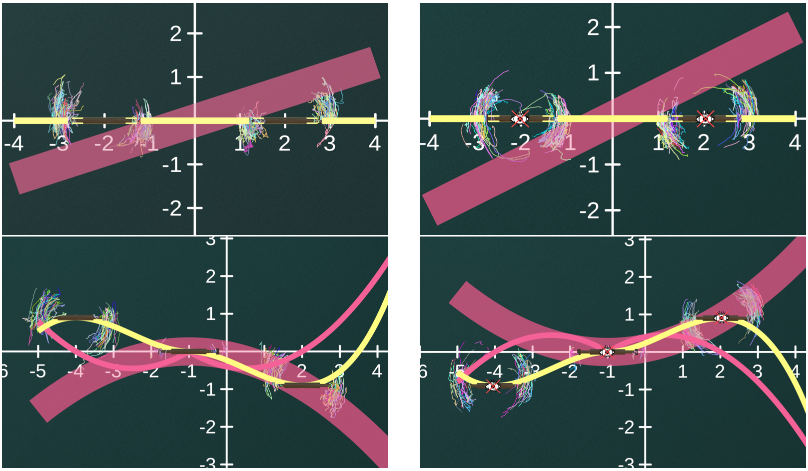


Figure 6.3: *Embodied analysis of students trajectories: Levels 15, 17, 20 and 21 in VR, zoomed in for clarity.*

- Level information: When a level start, end, and with which score
- Interaction information: When an interaction with a certain handle start, end, as well as intermediate states including handle state and hands' positions
- Position information: Where the user is located in the scene at each point in time

In this work, we focus on hand movements and decoupled behavior into two aspects: interaction and movement. Interaction is focused on the goal of bodily actions, for example, manipulating a handle. In contrast, movement is focused on the bodily actions themselves, for example, the speed and amplitude of the interaction. We decided to separate these two aspects as our previous work shows that manipulation and embodiment lead to different learning outcomes (Chapter 5 or [Chatain et al., 2023c]).

To evaluate interaction, we considered the number of interactions with each handle, particularly which handles were interacted with, as well as the sequence of these actions. To evaluate movement, we considered the angular amplitude of the movement as well as the sequence of such amplitudes. This is based on previous research highlighting that when communicating about functions' graphs, high-achieving students perform coarse movements while hard-working but low-achieving students perform finer movements [Gerofsky, 2011]. We first removed movements smaller than 1 degree as jittery movements due to tracking limitations. We then defined the threshold between small and large movements by using the median of movement amplitudes for each condition, which is 5.41 degrees. Importantly, we separated each interaction into several movements by a change of angular direction. This accounts for students who did not release the handle between different attempts towards the solution.

Finally, for both of these approaches, we also considered long idle states as a proxy for states of reflection. We selected a floor threshold of 5 seconds for such reflection states. This corresponds to the beginning of the right tail of the normal distribution of the base 10 logarithm of duration of idle states. This value is slightly lower than values used in previous work, for example, 8 and 10 seconds [Shih, 2011; Sinha and Aleven, 2015]. This is justified by the fact that our activity is faster paced than the ones used in these works. A summary of the events used in the sequences is presented in Table 6.2.

Table 6.2: *Description of the nodes used in the sequence analysis.*

Interaction	
INTX	The learner is interacting with the handle in position x.
Movement	
SMALL	The learner's hands are performing an interactive movement of small amplitude.
LARGE	The learner's hands are performing an interactive movement of large amplitude.
Other	
REFL	The learner is idle for over 5 seconds, this is used as a proxy for reflection.

Behavior analysis

For all the analyses, we discarded level 0 as this level was meant as an interaction tutorial.

For our numerical metrics, we performed a correlation analysis. When considering number of states (as defined in Table 6.2), we used Kendall's τ as the data presented a large number of tied ranks. When considering numerical values, we used the Pearson's r.

For our sequential metrics, we performed a sequence analysis focusing on maximum contrasts. For example, to address RQ1, one analysis focused on performing a sequence analysis with two clusters, one composed of 25 % of students with highest math anxiety, and one composed of 25 % of students with lowest math anxiety. Specifically, we ran a data-driven sequence analysis using the Markov based approach implemented with the `clickstream` package in R [Sinha and Aleven, 2015; Scholz, 2016; Ching and Ng, 2006]. With this approach, we evaluated the transition matrices between the states described in Table 6.2, focused on interaction or movement. To evaluate behavior evolution, we looked into four levels in particular. We used levels 10 and 11 to evaluate initial behavior as these are the first levels with several handles. Both levels are of similar difficulty. Level 10 is implemented with normal feedback while level 11 is implemented with delayed feedback. Similarly, to evaluate end-of-game behavior, we considered levels 20 and 21.

6.3.3 Results

In this section, we present the results of our analysis for each metric of interest, focusing on interaction (Figure 6.4) and movement (Figure 6.5). For clarity, we only include relevant results.

Math anxiety

For the TAB condition, we found a positive correlation between math anxiety and the number of interactions ($p < 0.001$, $\tau = 0.29$). This tendency existed for all movements: there was a positive correlation between math anxiety and the number of small movements ($p = 0.002$, $\tau = 0.27$) as well as the number of large movements ($p = 0.018$, $\tau = 0.21$). This means that participants who were more math anxious required more manipulations to reach the solution. In contrast, for the VR condition, we found no significant correlations between math anxiety and interaction, nor between math anxiety and movements.

This difference translated to post-test scores. In the TAB condition, we found a negative correlation between math anxiety and post-test scores ($p = 0.004$,

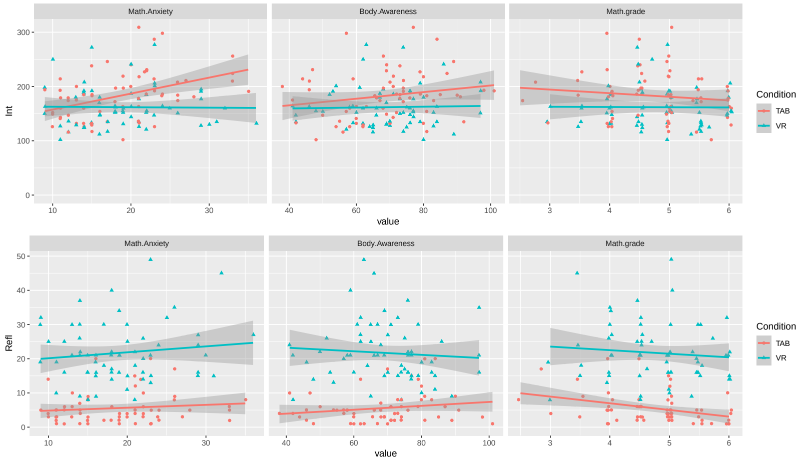


Figure 6.4: *Relation between individual differences and number of interaction events and number of reflection events. The* VR *condition is represented with blue triangles, the* TAB *condition with red circles.*

$r = -0.36$). In the VR condition, this correlation was not significant ($p = 0.11$, $r = -0.020$).

In conclusion, we found an effect of math anxiety in the TAB condition but not in the VR condition. Previous research showed that immersive technology could increase motivation for highly math-anxious individuals and positively impact learning [Chen, 2019]. Although we believe that motivation might have played a role in our result, we do not believe it is the main explanation, as we found no difference in the sequence of events between highly math-anxious individuals and their counterparts. In contrast, we believe that our results are due to the fact that, through its immersive properties, VR reduces access to social context cues, and therefore reduces social comparison and provides a safe space for exploration [Walker et al., 2021]. Indeed, part of math anxiety is due to social comparison [Dowker et al., 2016; Cipora et al., 2022], and in VR students could experiment with the content without fearing being judged by their peers or teacher.

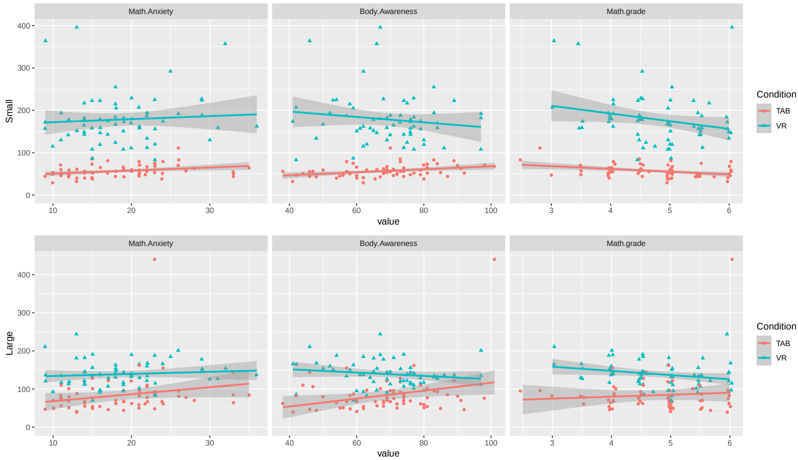


Figure 6.5: *Relation between individual differences and number of movement events. The* VR *condition is represented with blue triangles, the* TAB *condition with red circles.*

Body awareness

For the TAB condition, we found a marginally significant positive correlation between body awareness and number of interactions ($p = 0.052$, $\tau = 0.17$). Specifically, we found a significant positive correlation between body awareness and the number of small movements ($p = 0.002$, $\tau = 0.26$). In contrast, for the VR condition, we found no significant correlations between body awareness and interaction nor between body awareness and the number of movements. Moreover, for both both conditions, we found no correlation between body awareness and the total amount of movement nor between body awareness and the sense of agency on the mathematical curve.

These results contradict our expectations: Before running our analysis, we hypothesized that students with higher body awareness would be able to benefit from the VR condition best and reach the solution in less steps. To understand these results, we used our embodied analysis tool to explore students' trajectories and observed that trajectories in the TAB condition appear overall less precise (Figure 6.6). As individuals with higher body awareness, such as dancers, rely more on proprioceptive information than visual feedback [Jola et al., 2011], we believe that, when in a less precise and low embodied environment, they failed to reach their desired position when using small movements and often needed to readjust afterwards. Specifically, highly body aware individuals suffered more from the unproductive gap between proximal and distal movement [Abrahamson and Bakker, 2016]. The fact that the number of large movements was not correlated to body awareness supports this theory as large movements are not impacted by precision.

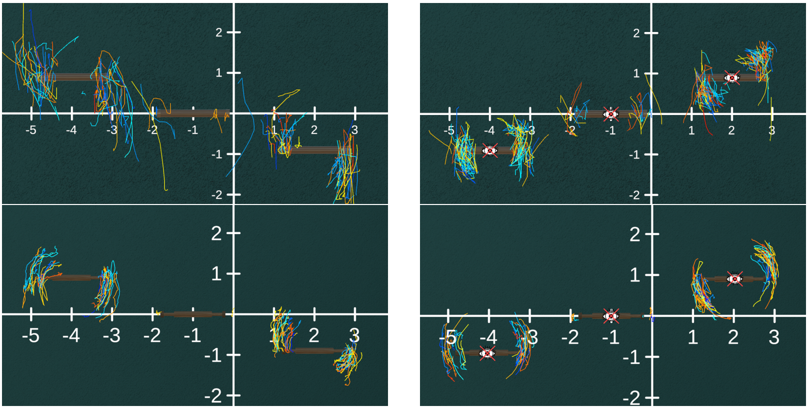


Figure 6.6: *Embodied analysis of high body awareness (reds) and low body awareness (blues). Levels 20 (left) and 21 (right) are represented, for the* TAB *(top) and* VR *(bottom) conditions. For clarity, the pictures are zoomed in, and the curves are not displayed.*

Math ability

For the TAB condition, we found a negative correlation between the grade of the participants and the number of reflection states ($p = 0.002$, $\tau = -0.23$), meaning that high achieving students needed less states of reflection to reach the solution, or thought less about how to solve the problem and deeper implications. Regarding movement, we found a negative correlation between math grade and the number of small movements ($p = 0.006$, $\tau = -0.25$).

In the VR condition, we found no significant correlation between math grade and interaction. However, we found a negative correlation between math grade and the number of small movements ($p = 0.026$, $\tau = -0.20$) and the number of large movements ($p = 0.018$, $\tau = -0.22$). This means that, although the number of interactions was not significantly lower for high achieving students, these interactions were composed of less movements.

Through the maximum contrasts sequence analysis, we found a behavior difference between students with highest math grades and lowest math grades, consistent across conditions. At the beginning of the game, students with higher grades followed a REFL ⟶ SMALL transition more often than REFL ⟶ LARGE (54% opposed to 43%). On delayed feedback levels, this tendency was similar although less pronounced (48% REFL ⟶ SMALL, 44% REFL ⟶ LARGE). However, by the end of the game, the tendency was reversed: events of reflection were followed by large movements in most cases (57% REFL ⟶ LARGE as opposed to 43% REFL ⟶ SMALL). This tendency was more pronounced on delayed feedback levels (67% REFL ⟶ LARGE, 33% REFL ⟶ SMALL).

In contrast, students with lowest math grades consistently followed events of reflection by small movements. At the beginning of the game, REFL ⟶ SMALL happened in 67% of cases, as opposed to REFL ⟶ LARGE in 33% of cases. On delayed feedback levels, the tendency was slightly reversed (47% REFL ⟶ SMALL, 53% REFL ⟶ LARGE). At the end of the game, the tendency was the strongest: 72% REFL ⟶ SMALL and 27% REFL ⟶ LARGE following moments of reflection in normal levels, and 64% REFL ⟶ SMALL and 36% REFL ⟶ LARGE on delayed feedback levels.

This illustrates that students with the highest grades had productive moments of reflections towards the end of the game, followed by a large movement to go closer to the solution, and subsequent smaller "fine-tuning" movements to increase the score. This is also illustrated by the fact that large movements were most often followed by small movements (59%). In contrast, students with lowest math grades used small movements after reflection, illustrating that reflection was unproductive and did not support error identification. This is also illustrated by the fact that large movements were most often followed by other large movements (55%).

In conclusion, high achieving students needed less reflection in the TAB condition. In both conditions, high achieving students needed less movements to reach the solution, and in particular less small movements. Moreover, across both conditions, students with higher math ability followed a "think, go to solution, fine-tune" behavior while students with lower grades followed a "think, try, iterate, fine-tune" behavior.

6.4 Spontaneous Bodily Actions Analysis

In the previous study, we evaluated the behavior of learners related to directed bodily actions. To complement this work, we now focus on sponta-

Table 6.3: *Demographics of interview participants.*

Id	Sitting?	Gender	Age	Math Level	Math Comfort	Teaching	Novice	Expert	Teacher
P1	No	Female	26	Secondary	2.00	No	Yes	No	No
P2	Yes	Male	26	PhD	5.00	Rarely	No	Yes	No
P3	Yes	Male	56	Masters	7.00	Daily	No	Yes	Yes
P4	Yes	Female	22	Masters	7.00	Weekly	No	Yes	Yes
P5	Yes	Female	22	Secondary	1.33	No	Yes	No	No
P6	Yes	Female	19	Bachelors	6.66	Rarely	No	Yes	No

neous bodily actions, that is, we study the bodily actions that participants spontaneously perform when communicating and reflecting about derivatives.

Through this qualitative study, we answer the following research question:

RQ2 Which bodily actions do individuals spontaneously perform when making sense of derivatives?

6.4.1 Demographics

For this qualitative study, we recruited participants with different profiles: novices in mathematics, experts in mathematics, and teachers of mathematics. The profiles of the recruited participants are summarized in Table 6.3. Math level describes the maximum level of education at which the participants used mathematics, math comfort is an average of the self-reported measures of positive math fluency, positive math affect, and negative math anxiety, on 7 points Likert scales. Based on this information, we label P1 and P5 as math novices as they have low math level and math comfort, P2, P3, P4 and P6 as math experts as they have high math level and math comfort, and P3 and P4 as math teachers as they have high teaching experience. There is an overlap between the set of teachers and the set of experts as math teachers are also math experts.

6.4.2 Protocol

The participants were interviewed online, individually, using a video conference software. The intervention included four tasks and a demographics questionnaire, and lasted around 30 minutes. At the beginning of the session, the researcher mentioned that the goal is to answer different questions around four tasks, and that, while communicating their answers, the participants are not allowed to use tools such as their mouse, pen or paper. Gestures were not mentioned explicitly, unless the participants specifically asked. The interviews included four tasks, targeting different scenarios where derivatives are relevant. The three first tasks targeted concrete relatable scenarios, and the last task focused on an abstract curve. For each task, the pictures were displayed on the participant's screen.

Slopes We first focused on slopes in relatable, visual, and embodied scenarios such as hiking. The users were asked to imagine that they were carrying a heavy backpack and had to describe which path would be most difficult to walk, assuming they start from the red dot and go to the green dot (Figure 6.7). We then asked them to justify their answer. This task emphasizes derivatives as slopes.

Figure 6.7: *Spontaneous bodily actions: Task 1 focused on derivatives as slopes.*

Speed With this task, we focused on speed, and used the trajectory of a rocket to support this exercise (Figure 6.8). The users were asked to describe how the speed of the model rocket behaves throughout the flight from launch until the descent. This task emphasizes derivatives as speed.

Variation rate With this task, we wanted to move towards abstraction and graphs of functions, while keeping the task relatable. As we looked for graphs that would be relatable for all participants, we decided to use the curves of cases of COVID-19 as a support for the task (Figure 6.8). We used two graphs: the graph of daily COVID-19 cases in Switzerland, and the cumulative graph of these cases, that is, for each day, the total number of cases until that day. This means that the first graph represents the derivative of the second graph. First, we asked the participants to describe the evolution of daily COVID-19 cases in Switzerland. We then asked them to explain whether the two graphs relate to each other, and how. This task emphasizes derivatives as variation rates.

Abstract With this final task, we wanted to evaluate a more abstract situation, e.g. the graph of a function as one would encounter in a math class (Figure 6.8). We selected a function including a local maximum, a local minimum, as well as an inflection point ($f(x) = 0.4x^5 - 1.5x^4 + 4x^2 - 1$). The participants were simply asked to describe the graph. This task emphasizes derivatives as mathematical objects.

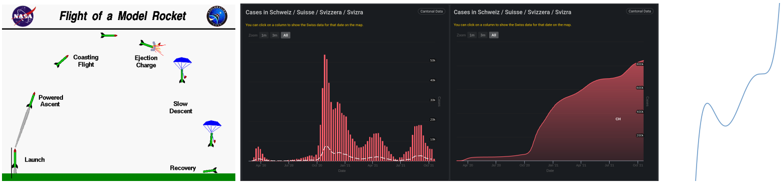


Figure 6.8: *Spontaneous bodily actions: Task 2 (left) focused on derivatives as speed, task 3 (center) focused on derivatives as variation rates, task 4 (right) focused on derivatives as mathematical objects.*

6.4.3 Results

We annotated the videos with both speech and gestures. We focused only on the gestures related to derivatives, and ignored body movements such as scratching or moving one's hair. We then analyzed the annotations using an inductive thematic analysis [Braun and Clarke, 2006]. This procedure was conducted by the first author. In total, we gathered 111 hand movements, 21 in task 1, 24 in task 2, 29 in task 3 and 37 in task 4. Specifically, P1 performed 24 hand movements, P2 performed 17, P3 performed 3, P4 performed 24, P5 performed 26, P6 performed 17. We believe that the low number of hand movements performed by P3 is due to the fact the P3 struggled to see the pictures on the screen and had to stay close to it in order to answer the questions. In the following, we detail the results of this analysis.

General and specific behaviors are described using different hand poses and trajectories Hand trajectories were mainly vertical and horizontal, rather than diagonal. Indeed, 35 hand trajectories were vertical and 37 were horizontal, while only 15 were in diagonal. Diagonal trajectories were mostly used by teachers (8 occurrences) and primarily in tasks 4 (6 occurrences) and 1 (4 occurrences). Diagonal trajectories were used in conjunction with a pointing hand pose and used to draw the curve precisely (10 occurrences). In contrast, vertical and horizontal hand trajectories were mostly used with flat or slightly curved hands. Vertical hand trajectories were used in conjunction with words such as ”increase”, “up”, “decrease” or “down”, while horizontal trajectories were used with words such as “plateau”, “flat”, or “stationary”. From this, we conclude that learners mostly focus on general up, down, and flat behavior, and do so using flat or slightly curved hand poses. When they need to be more precise, for example to compare different slopes in task 1, or describe a specific shape in task 4, they use a pointing gesture to draw the slope precisely (Figure 6.9).

Figure 6.9: *P5 uses a flat hand moved vertically to describe an increase; P2 uses a flat hand moved horizontally to describe a flat point; P3 uses a pointing gesture to describe a specific slope in diagonal.*

Steepness is represented with hand tension Although most gestures are either vertical or horizontal, and the steepness is therefore not always expressed in hand trajectory, we noticed that participants used hand tension to express steepness. For example, a tensed hand, that is a flat hand, with all fingers connected, is used to represent higher steepness and in conjunction with words like “large”, “strong”, “sharp” or “very”. In contrast, a relaxed hand, that is with slightly curved fingers, a bit spread apart, is used to represent lower steepness and in conjunction with words like “small”, “less” or “slight”. For example, this was the case to represent mountains as opposed to hills in task 1, or steep variations as opposed to slower variations of COVID-19 cases in task 3 (Figure 6.10). Sometimes, we observed a mismatch between speech and gesture [Roth, 2001]: For example, on task 3, P1 kept using words such as “strong” and “very” while describing the later peaks of cases, but used a more and more relaxed hand to support her point. In this case, her gestures described the situation accurately, but her speech was as strong for all the peaks. We believe that this shows that P1 understood the global structure of the graph at the embodied level, but focused on the local perspective at the speech level. From this, we conclude that learners may use hand tension to express steepness, over hand pose and trajectory. We also note that, as described in previous work, a mismatch between gesture and speech should be considered to identify first evidence of learning [Roth, 2001].

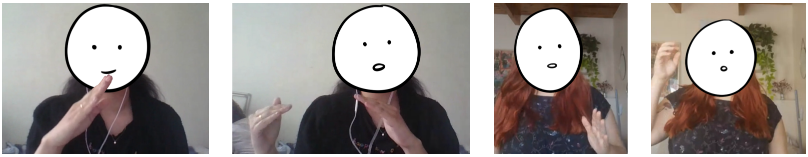


Figure 6.10: *P5 gestures a steep mountain with a tensed hand and a hill with a relaxed hand; P1 gestures a steep increase with a tensed hand and a less steep increase with a more relaxed hand.*

Repetition is used to emphasize characteristic behavior Participants, novices in particular, often used repetition, that is repeating a certain hand movement several times in a row. Specifically, 37 hand gestures used repetition, most often along the horizontal axis (20) and the vertical axis (10). Out of these, 21 were performed by novices. Repetition was most often used to anchor a high level reasoning, either by presenting a characteristic point of a curve or by presenting a key element of the situation. For example, peaks and plateaus were often described using repetition: "The speed stabilizes itself at the top" (P6), "It reaches its peak" (P2), "Then you have plateau" (P4). Moreover, high level reflections are anchored in repetition: "I think there is a bit of distance to walk, it doesn't go up a lot, but it goes for a long time" (P1), "I believe this is why we kept such a low level of corona virus cases" (P6), "There's not many trees or forest around either" (P5). In conclusion, learners might use repetition to emphasize characteristic aspects and deep features of a certain problem or situation. This is particularly important to identify in PS-I pedagogical patterns as recognition of deep features is an important mechanism of learning in this context [Loibl et al., 2017].

Reference points are anchored in space Some participants used space to anchor reference points and describe a certain behavior in reference to that point (Figure 6.11). For example, on task 4, P4 first gestured the entire curve, and then started explaining which terms might compose this curve. She hypothesized that, because of the plateau and the high increase at the end, a square term and a cubic term were part of the formula, and positioned them on different points of the curve. In contrast, P6 used explicit anchors for her reasoning. As it is often the case in sign languages, P6 often used both hands, using one as a passive, anchor hand, and one as an active hand.

To describe a slope, P6 kept the passive hand at a position of reference, and moved the active hand to draw the slope. This means that learners' gestures should be considered within their spatio-temporal context.

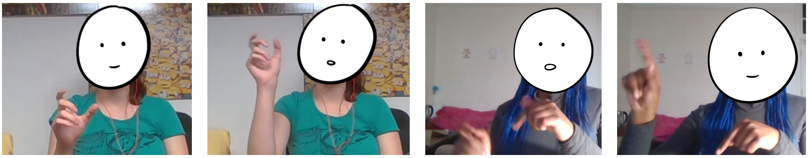


Figure 6.11: *After gesturing the general shape of the curve, P4 explains where the influence of each term is visible; P6 uses a finger as an anchor to describe a specific slope.*

Novices use a first person perspective Although most participants used a third person perspective, that is looking at the situations as an external observer on the side, P5 often used a first person perspective, that is considering that she was actually standing on the curve (6 out of 26 hand movements). This behavior was consistent across all tasks. Although we did not find evidence of this perspective with P1, the other novice participant, we believe that this is an important aspect to consider. When derivatives are experienced outside of the classroom, at the embodied level, they are often experienced from a first person perspective, for example in a plane or a roller coaster. Although some first person embodied input systems to experience derivatives exist [Tancredi et al., 2022], there is no VR activity including this perspective both at the input and visualization level, and we believe this would be an interesting direction to explore in the future. Moreover, a broader conversation regarding the role of technology on perspective is required. Indeed, as described in Chapter 4 and Chapter 5, the tablet implementation of the embodied activities supports the third person perspective. In turn, VR activities tend to support the first person perspective as the rendering of the virtual scene is usually aligned with the user position. However, this aspect is under-explored and could be used to support stronger embodiment by not only aligning the perspective of the user with the one of the digital avatar, but also with the mathematical object itself.

Embodied metaphors are spontaneously integrated Participants also used gestures to mimic specific elements of their description in the more concrete tasks (Figure 6.12). For example, on task 1, participants used their hands to mimic the mountains and hills (P1, P4, P5, P6). Similarly, on task

2, all participants mentioning the parachute also used gestures to represent this parachute during the fall of the rocket (P4, P5, P6). Moreover, on task 3, all participants mentioning a step by step increase also mimicked stairs (P2, P4). This means that, especially when the content is grounded in concreteness, learners might rely on concrete embodied metaphors to express their understanding, rather than directly gesturing the underlying mathematical behavior at a more abstract level.

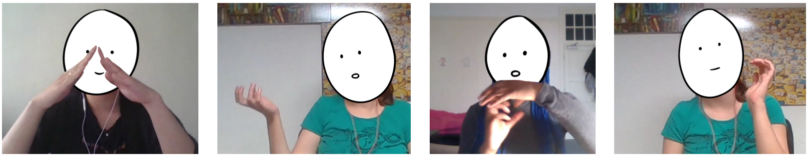


Figure 6.12: *P5 gestures a mountain; P4 and P6 gesture a parachute; P4 gestures stairs to describe a gradual increase.*

6.5 Discussion

This chapter aimed to understand how learners use bodily actions in sense-making tasks and draw conclusions on how to design embodied learning activities. To do so, we performed two analyses. In the first analysis, we focused on directed bodily actions in intuition-building tasks on the topic of derivatives. In this context, we explored the role of individual differences in bodily actions. We focused on spontaneous bodily actions in a set of intuition-probing tasks on derivatives, in the second analysis. This study identified key characteristics of bodily actions performed while reflecting and communicating derivatives.

In this section, we detail our resulting design recommendations, and describe limitations of our work as well as directions for future research.

6.5.1 Design recommendations

From both analyses, we assembled design recommendations for future VR tools for embodied mathematics sense-making.

Expand embodied interaction design beyond position and movement
Although sensorimotor simulation is an important mechanism of embodiment, it is not the only one [Körner et al., 2015]. Direct state induction

highlights the fact that different body states, for example muscle tension, result in different feelings. This aspect has also been explored in embodied interaction research through the distinction between the flesh body, *Körper*, and the feeling body, *Leib* [Mueller et al., 2018]. Designing for the *Körper* reduces learners as physical entities utilized to press buttons and perform actions. Designing for the *Leib* acknowledges learners as feeling entities and benefits from direct state induction. Such considerations are also important to reduce math anxiety as *Leib*-informed poses, such as a power pose, could boost confidence [Isbister et al., 2012], and offer a sense of embodied achievement [Chatain et al., 2022; Chatain et al., 2023c]. In our second study, we noticed that participants relied on muscle tension to express steepness. This illustrates the fact that learners spontaneously rely on direct state induction, rather than solely position and movement. Therefore, we recommend considering the *Leib* perspective when designing embodied interaction, and its relation to mathematical meaning. Additionally, body-centered approaches such as somaesthetic appreciation design could be considered [Höök et al., 2016].

Consider embodied metaphors and embodied concreteness Another mechanism of embodiment is modal priming [Körner et al., 2015]. Modal priming relies on conceptual metaphors to ground mathematics in concreteness [Lakoff and Johnson, 2008]. Our second study showed that learners spontaneously use embodied metaphors when making sense of mathematical content, specifically in concrete contexts, by gesturing mountains, hills, or a parachute, but also in more abstract contexts, for example by gesturing stairs to describe a gradual and discrete increase. Novel input mechanisms from Human-Computer Interaction research could be used to support such metaphors at the interaction level. For example, the digital glove mechanism combines interaction and display on the hands of the users [Chatain et al., 2020], and hand interfaces transform users' hands into tools based on embodied metaphors [Pei et al., 2022]. Expanding beyond the hands, our study also revealed that learners use different perspectives when making sense of derivatives: some learners used a third person perspective, observing the curves from the side, while other learners used a first person perspective, positioning themselves on the curve. Considering that most embodied experiences are lived from a first person perspective, we recommend exploring a first person perspective when designing embodied learning activities. Such approach aligns with a design centered on embodied concreteness, that is "a form of concreteness that involves a high degree of embodiment, in a situated and relatable context", and can be powerful for grounding abstract mathematics [Chatain et al., 2023c]. Different VR viewpoints could also be considered [Galvan Debarba et al., 2017].

Allow coarse gesturing for identification of deep features Previous work highlighted the importance of aligning interaction technique precision with the accuracy requirements of the activity [Chatain et al., 2022]. However, the accuracy requirements of the activity were not discussed. Our first study revealed that students with higher math ability performed less small movements. In turn, our second study highlighted that learners used coarse movement and focused more on general behavior of the curves rather than specific behavior, especially along the x axis, for increasing and decreasing behavior, and the y axis, for plateau behavior. Moreover, learners used repetition to highlight characteristic behavior and deeper reasoning. More generally, coarse movement is often used by teachers and students with high math ability to communicate about functions' graphs [Gerofsky, 2011]. We recommend supporting coarse movement and repetitions in embodied learning activities. For example, the embodied learning activity used in our first study uses gamification to provide feedback to the learner. Specifically, it provides a real-time score between 0% and 100% to convey how close to the solution their curve is. This score can negatively impact embodied sense-making as it emphasizes accuracy and coarse or repetitive movements would result in a temporary score reduction.

Support and evaluate sense-making anchors When designing digital solutions for embodiment, we ought to consider two types of movements: proximal movements and distal movements [Abrahamson and Bakker, 2016]. If we consider the interactive components of an activity as instruments, proximal movements describe the movements performed to interact with the instrument, while distal movements describe the effect of said instrument on the world. Our first study highlighted that imprecision, which creates a gap between proximal and distal movement, is penalizing for highly body aware learners. However, such gap can also be productive, for example as a case of desirable difficulty [Abrahamson and Bakker, 2016; Chatain et al., 2022; Bjork et al., 2011]. Specifically, the interaction should support the creation of attentional anchors, that is imagined and spatially located instruments to support sense-making [Abrahamson and Bakker, 2016]. In our second study, we observed that participants relied on spatial anchors to illustrate attentional anchors, for example by using two fingers to draw an imagined slope or relying on previous movements to locate new gestures. To support the creation of attentional anchors we recommend considering which elements of the mathematical objects should be visible, and which should be imagined. For example, in our first study, the slope is represented as a handle on the function curve and visible at all times. A similar activity without the handle might create more space for attentional anchors. In turn, to evaluate the creation of attentional anchors in embodied VR learning activities, we recommend tracking hand movement during reflection times rather than solely during interaction. This is particularly important as, in our first study, students with higher math ability performed less interactive movements, and, instead, used productive moments of reflection to reach the solution. Eye-tracking technology can also be considered for this purpose [Abrahamson et al., 2015].

Integrate embodied in-VR learning assessments Our results showed that, unlike its tablet counterpart, our highly embodied activity in VR did not show significant effects of math anxiety on interaction nor on post-test scores: VR provides a safe space for exploring and learning mathematics [Walker et al., 2021]. In this light, we question the relevance of disembodied and out-of-VR learning assessments. When making sense of novel content, learners are able to express their understanding in gestures, before they are able to articulate it in speech or in writing [Roth, 2001]. Therefore, focusing only on written assessments and interviews is not enough to catch first evidence of preliminary learning. Moreover, previous work has shown that cognitively relevant gestures can impact performance provided that learners actually used gestures during subsequent assessment [Walkington et al., 2022]. This can be particularly problematic as we noticed in our first study that students with higher math ability performed less movements than students with lower math ability and thus might benefit less from the embodied learning activity. Therefore, disembodied learning assessments penalize math anxious individuals, prevent identification of preliminary understanding, and reduce the effect of embodied learning altogether by limiting gesture production. In alignment with design principles for embodied VR learning activities [Johnson-Glenberg, 2019; Tran et al., 2017], we recommend considering embodied in-VR learning assessments. We believe that such assessments would greatly impact embodiment research by offering adequate tools to evaluate learning outcomes. To address specific design considerations related to in-VR assessments, we recommend general literature on VR questionnaire design [Safikhani et al., 2021] and on VR mathematics input interfaces design [Sansonetti et al., 2021].

6.5.2 Limitations and Future Work

There are several limitations to this work. First, the activity used for our directed bodily actions study focused on one interaction technique specifically, the two-hands handle approach, and therefore highly constrained the exploration space. We believe that this limitation does not invalidate our work as we chose this interaction technique following previous work on embodied interaction for derivatives [Chatain et al., 2022]. Moreover, we conducted an internal study with $n = 33$ participants where we compared this interaction technique to other techniques based on the results of our spontaneous bodily actions study. Specifically, we compared the two-hands handle interaction technique to a one-hand position technique and a one-hand velocity technique. Our results showed that the two-hands handle approach is significantly more usable and more intuitive than the alternatives. Nevertheless, future work should explore the link between individual factors and bodily actions in other interactive contexts.

Another potential issue is that our directed embodied activity included elements on gamification. Indeed, a score was displayed on the screen, updating the student in real time on how well they performed. This impacted behavior as some students were focused on reaching a 100% score rather than making sense of the problem at hand. We believe this issue is mitigated as we conducted an analysis of our data using video game experience as an independent variable and did not find any differences in our dependent variables. However, we expect an activity based explicitly on exploration and sense-making to raise different results and believe this should be explored in the future.

The directed bodily actions study presents other limitations. First, due to time constraints and ethical concerns, we used self-reported math grades as a proxy for math ability. This is not optimal as students might wrongly report their math grade. We believe that the self-reporting issue was partially mitigated as many students used their phones to retrieve their grades, but we do not have a way to assess the impact with certainty. Moreover, math grade covers a wide range of topics, and we believe that a more specific assessment would be beneficial. Another issue with this study is the length of the intervention. Indeed, we believe that repeated sense-making sessions over a longer period of time would enable future researchers to evaluate the behavior evolution from novice to expert. Sense-making is a process of several steps [Davidson et al., 2022], and our intervention was too short to cover this progression. This is a general issue in the field of embodiment research [Ale et al., 2022]. Finally, all participants were recruited from the same area of Switzerland. Although this is a highly international area, we do not believe that our study is sufficient to account for cultural diversity and its impact on bodily actions.

Regarding our spontaneous bodily actions study, the main limitation is the low number of participants as well as the lack of cultural diversity. Although we recruited participants from diverse countries and backgrounds, five participants are not enough to account for all possible perspectives and approaches. However, as part of our exploratory work, we believe that our results already present several avenues for future work and highlight the gap between design for directed bodily actions and design for spontaneous bodily actions. However, we believe that future studies with more participants would help highlight other interesting research directions. For example, future work could investigate the role of attentional anchors in resulting spontaneous gestures in sense-making contexts [Abrahamson and Bakker, 2016]. Moreover, future work should explore spontaneous gestures production across a panel of users with diverse bodies [Spiel, 2021; Keehner and Fischer, 2012]. Indeed, different individuals move so differently that it is even possible to identify them through hand tracking data [Liebers et al., 2022].

6.6 Conclusion

In this work, we explored bodily actions performed while making sense of mathematics, and in particular derivatives. First, we conducted a quantitative user study where we evaluated the role of individual differences on directed bodily actions. This study revealed that VR provides a safe space of math anxious individuals, that highly body aware learners are penalized by unproductive gaps between proximal and distal movements, and that students with higher math ability perform less movements and rather rely on productive reflection times. In our second study, we focused on spontaneous bodily actions and performed a qualitative analysis to identify key characteristics of such bodily actions. This study revealed that learners use coarse movements and focus on general behavior when describing derivatives, use hand tension to represent steepness, emphasize characteristic behavior with repetition, anchor reference points in space, may use first person perspective over third person perspective, and integrate embodied metaphors.

From both of these studies, we identified design recommendations for future work. Specifically, we recommend designers expand embodied interaction design beyond position and movement, consider embodied metaphors and embodied concreteness, allow coarse gesturing for identification of deep features, support and evaluate sense-making anchors, and integrate in-VR learning assessments.

With this last work, we complete our exploration of the design landscape of embodiment, specifically in the context of mathematics. Moreover, we hope to open new avenues of research in the field of embodied interaction design, specifically for sense-making of mathematics.

Implications

In this section, we discuss the results from our projects within the same framework, offer directions for future work based on this discussion, and conclude.

7.1 Discussion

In this work, we addressed the following question:

RQ How to design embodied interaction to support embodied sense-making of mathematics?

To achieve our goal, in Chapter 2, we started by defining the system representation of embodiment that our work operates in. We then explored this framework through four starting points: avatar, interaction, context, as well as learners together with bodily actions. In Figure 7.1, we present the space covered by our research. Importantly, we refrain from highlighting "embodied interaction" fully as, although we explored the physical context of the interaction, we did not explore its social context.

In the following, we present take-away messages, informed by our work, and relevant for the design of future embodied learning activities. We consider the perspectives explicitly acknowledged in this work: learner, avatar, bodily actions, interaction, and context. Additionally, we mention results specific to Virtual Reality (VR) technology itself.

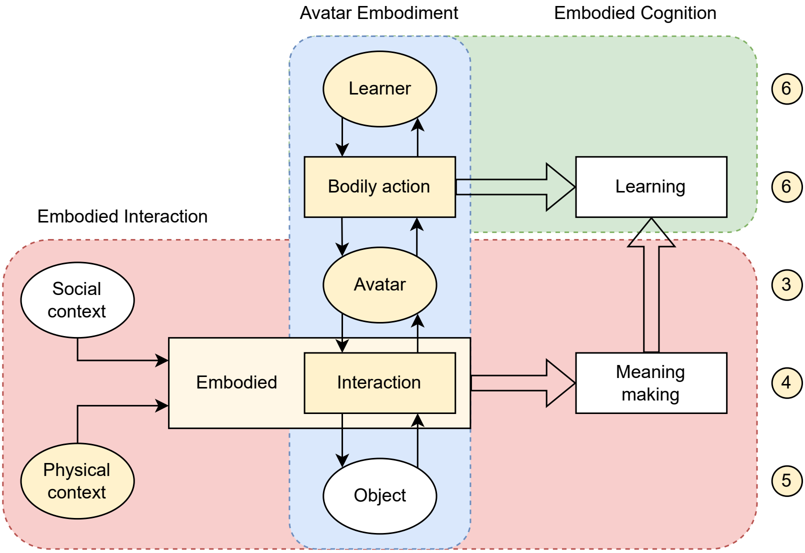


Figure 7.1: *System representation of the embodiment landscape this thesis operates in. The starting points explored in the work are presented in yellow. The related chapters are presented as annotations on the right.*

Digital avatars can be used to create meaning. Embodied learning can happen prior to interaction: bodily actions without an explicit object target can also support embodied learning [Melcer and Isbister, 2016]. For example, counting on one's fingers can help one learn about basic arithmetic. Moreover, through their embodied exploration of the world, humans learn how to use their hands and body to communicate and anchor meaning, often spontaneously [Roth, 2001]. In the virtual world, users get to learn about a new world and explore it through a digital avatar [Kilteni et al., 2012]. Previous work highlighted that avatar design can impact cognitive performance [Banakou et al., 2018] as well as behavior [Kilteni et al., 2013].

In Chapter 3, we show that avatar design can be used to make certain gestures meaningful. For example, in the physical world, a pinch gesture has little meaning. However, when displaying a world on one's hand, a pinching gesture clearly means creating a passage between two parts of the world. This idea can be expanded further. We can design what we call "semantic avatars": digital avatars that are designed to highlight a specific meaning, and thus support intuition building otherwise out of reach with physical bodies. For example, creating avatars with four fingers per hand to support base 8 finger counting, or avatars with stretchable arms to make sense of linear algebra in 2D and 3D spaces (Figure 7.2).

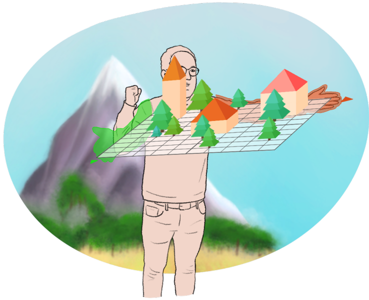


Figure 7.2: *Example of full-body semantic avatar: Using an avatar with stretchable arms learners could embody a space's referential and learn about 2D linear algebra.*

This approach is also important and interesting to support the diversity of bodies often ignored in embodiment research [Keehner and Fischer, 2012; Spiel, 2021]. We can design and generate digital avatars specifically suited to give each individual access to embodied experiences that are usually not reachable with their bodies. For example, in Chapter 4, we presented a two-handed interaction to rotate a handle representation of the derivative. This interaction did not work for one-handed participants. To solve this issue, we could provide an avatar with longer fingers to rotate the handle while not losing visibility.

Embodied interaction does not necessarily support embodied cognition. An important result of our work, is that not all embodied interaction support learning, or at least not with the same magnitude. For example, in Chapter 4, we showed that a movement-focused approach to interaction impaired learning and persistence as opposed to a position-centered approach. In addition, in Chapter 5, we showed that, as opposed to bodily manipulation only, embodiment supports grounding and does not impair transfer. From this, we conclude that separating embodied interaction from embodied cognition is a useful and necessary approach when designing embodied learning activities. In our framework (Figure 7.1), we highlight two paths to learning: from bodily actions themselves, and from interaction through meaning-making. Importantly, meaning-making, here, refers to the act of giving meaning the interaction [Dourish, 2004]. Previous work also highlighted the importance of disconnecting the bodily actions from their effect on the world [Abrahamson and Bakker, 2016]. It is in this gap between bodily actions and interaction that productive opportunities for cognition can be identified.

This has two main implications for interaction design. In Chapter 4, we highlighted the importance of considering the mathematical meaning emphasized by different interaction techniques. For examples, we hypothesized that a position-based approach to interaction highlights derivatives as slopes, while a movement-based approach focused on the variation rate interpretation. Therefore, using a movement-based approach in a task focusing on derivatives as slopes may be counterproductive as making sense of the interaction in that case is not aligned with sense-making of the mathematical object. Previous work has argued that congruent gestures are of critical importance when designing embodied learning activities [Johnson-Glenberg and Megowan-Romanowicz, 2017; Nathan and Walkington, 2017]. In this situation, congruence refers to an alignment between movement and mathematical concept. Our work provides a finer distinction as it shows that congruence between interaction meaning and mathematical meaning in the context of the learning activity also matters.

Another implication is that meaning-making often happens in the invisible. In Chapter 6, we showed that learners use spatial anchors to support reasoning. After giving meaning to a certain point in space, they may express new elements in reference to this point, either in space or in time. Using eye-tracking in a two-handed activity about proportions, Abrahamson and Bakker also demonstrated that learners' focus was not necessarily on their hands, but rather on the space between their hands, as if they were relying on invisible and imagined attentional anchors [Abrahamson and Bakker, 2016]. When designing embodied learning activities, especially in VR, it is tempting to make mathematical objects as visible and perceptible as possible. However, giving space for the learners' to create these spatial and attentional anchors themselves can actually be beneficial for embodied cognition.

This space can be created in many ways. First, as mentioned before, designers can create this space by introducing distance between interaction and its result. This might require sacrificing usability to implement desirable difficulties, such as our *delayed-feedback* levels in Chapter 4. Second, distance could be created between the learner and the content and induce a need for productive indexing [Hornecker, 2016]. For example, in our derivatives prototype, this could be done by placing the function curve and the derivative curve such that both cannot be perceived in one gaze. Third, by introducing a reason for communication beyond the content. Collaborative tasks are particularly suitable for this, and can benefit for game-based approaches. For example, in the game "Keep Talking and Nobody Explodes" two players have to diffuse a bomb [Games, 2015]. One player is handed in a manual with instructions, including abstract symbols, to diffuse the bomb while the other player can manipulate the bomb. As either player cannot access the other player's information directly, they have to communicate, and converge towards a common language to communicate efficiently before the bomb's countdown. Such approach would be particularly beneficial in mathematics learning as mathematical symbols were created for exactly this: communicating.

Embodied interaction is tied to expectations from the physical world. Our work also shows that embodied interaction is informed by the physical world. This is not very surprising as this aspect is at the core of the definition of embodied interaction [Dourish, 2004]. However, what is new in our work, is the understanding that this aspect is robust to immersive virtual environments. In Chapter 3, we noticed that, as we placed users' bodies at the core of the digital activity, participants brought expectations from the real world, even if these expectations made the interaction more cumbersome of barely usable. For example, one of our games included a twin-pan balance gesture, with the hands slightly tilted to improve visibility of the content. However, several participants kept their hands perfectly flat, and, even when reminded that they could tilt the hands to see better, kept switching back to the horizontal gesture while simultaneously complaining about visibility. This is particularly important when using hand tracking over controllers. Indeed, while controllers introduce flexibility by their specific usage, learners are used to interacting with the world using their bodies and hands and will struggle to distance themselves from their habits. They will expect their bodies to have similar effects on the virtual world.

As hand tracking is often better from an embodiment perspective, it is important to bear in mind that in comes with this set of expectations. There are two approaches to deal with this. First, a solution is to include a gesture training both for the human and the machine. For the human, gesture training is used to re-adjust expectations and give space for getting familiar with the novel approach. For the machine, this training can be used to identify robustness to change but also to calibrate the hand tracking algorithm to the diversity of hand movements and poses, as identified in Chapter 3. Second, another approach would be to identify these expectations and use them beneficially for interaction design. For example, hand interfaces are hand input techniques imitating real life objects, such as scissors or binoculars [Pei et al., 2022]. In Chapter 6, we concluded that learners used metaphorical gestures to support their argumentation. Identifying such gestures and integrating them in embodied learning activities at the interaction level would be an interesting direction to explore.

Embodied interaction creates a shift in perspective. In Chapter 6, we identified that when making sense of derivatives novice learners may use first perspective spontaneous bodily actions over the more common third perspective approach. Concretely, if we imagine a function's curve as a roller coaster, these students would imagine themselves riding the roller coaster rather than observing it from the side. The question of perspective is an important one in embodiment research. In the physical world, although vicarious embodiment can be argued for, most embodied experiences are perceived from a first person perspective. Therefore, embodied learning activities in VR should account for the first person perspective.

We followed this approach in Chapter 5, in our embodied concreteness condition where students could manipulate a pipe-system, from a first person perspective, to reason about graph theory. In the preliminary usability study where we compared this approach to a similar approach on tablet, we noticed that the perspective shift could be problematic as participants in the tablet condition could access an overview of the problem immediately. To solve this issue, we included an overview of the problem in the first-perspective condition as well. This can be problematic as it requires learners to navigate between two representations. However, this can also create a productive gap for sense-making and support positive indexing [Hornecker, 2016].

We believe that this direction should be explored in more depth, either by using different perspectives simultaneously, as in our project, or by integrating a sequence of perspectives, for example by starting from a first person perspective and then being able to vicariously experience the same scene.

Embodied interaction needs to support direct state induction. The mechanisms of embodiment go beyond sensori-motor simulation [Körner et al., 2015]. Direct state induction refers to the idea that certain body position and movements directly induce certain states in users or learners, before any cognitive process. For example, holding one's shoulders high while keeping the head down induces a state of tension. Proponents of embodied interaction have argued that considering bodies as feeling entities, so-called *Leib*, should be the main paradigm in embodied interaction design [Mueller et al., 2018]. We support this direction in two ways.

In Chapter 6, we observe that learners spontaneously rely of direct state induction, for example by representing slopes of higher magnitude through hand tension. Observing such spontaneous processes can greatly inform design and offer novel solutions for interaction. For example, we mentioned earlier the possibility of creating a digital avatar with stretchable arms to support sense-making of linear algebra. However, human arms are usually not stretchable. A simple solution to this problem is to keep stretching the arms of the digital avatar based on a certain hand gesture, e.g. opening and closing the hand. However, another approach could be to consider arm tension directly, and keep stretching the arm as it remains tensed, and stop the stretching as the arm relax, up to a point where the arm progressively goes back to its initial length.

Moreover, in Chapter 4 and Chapter 5, we used direct-state induction by designing a form of embodied achievement. In our derivatives game, learners are invited to "high-five" a button to switch to the next level (Figure 7.3). In our graph theory game, the switch is performed by adopting a winning position, raising both arms in the air. Although we did not rigorously evaluate the impact of this approach on learners' experience, we received a lot of positive feedback and observed that learners adopted a playful attitude during the interaction, for example by exclaiming "High five!" or laughing.

Figure 7.3: *In our derivatives game, a student "high-fives" a button to go to the next level and gets a sense of embodied achievement.*

Interaction context supports grounding through modal priming. Going beyond the interaction itself, we also explored the role of interaction context. In Chapter 5, we looked at this question through the lens of modal priming. Modal priming refers to the mechanism through which specific sensorimotor states bring certain concepts to mind, often through conceptual metaphors [Körner et al., 2015]. More concretely, we explored how embodied metaphors, that is metaphors informed by our embodied experiences of the world [Lakoff and Johnson, 2008], can be used to facilitate modal priming from a concrete embodied experience towards more abstract concepts of graph theory. To do so, we used embodied interaction within an embodied metaphor of a graph as a pipe-system. The goal of the learners was then to bring the maximum amount of water from a lake to a city.

In this work, we identified that our game supported grounding of graph theory without impairing learning outcomes. Through an in-depth discussion about the mechanisms involved in this process, we identified that modal priming is the main mechanism justifying the benefits of our approach for grounding. Therefore, we conclude that this mechanism should be explicitly accounted for when designing embodied learning activities. Embodiment is not just about involving learners' bodily actions in the activity, it is also about designing a relevant and meaningful context for these actions.

VR can support embodied interaction after a period of familiarization. In our work, we focused on VR as a technology to implement embodied learning activities. Although, as demonstrated in Chapter 2, VR offers many affordances for such activities, it also comes with certain drawbacks identified in our work.

In Chapter 4, we noticed that VR can reduce bodily movements in two ways. First, learners might be concerned about bumping into tangible elements from the real world, and injure themselves or others. Second, because this technology is rather recent and most people do not have experience with it, some users might be hesitant to perform certain actions because they do not know what is possible and what is not.

These issues can be addressed by letting users gain experience with the system over more longitudinal approaches. However, a short VR familiarization phase can also be helpful. We included such familiarization step in our activities, both in Chapter 4 and Chapter 5, and observed that it helped learners feel more confident with the activity.

Such VR tutorial should include two phases, in order to account for the two issues we identified (Figure 7.4). First, a phase where students can walk around the space, and identify its boundaries. Second, a space where learners can interact with virtual objects, independently of the subsequent learning activity.

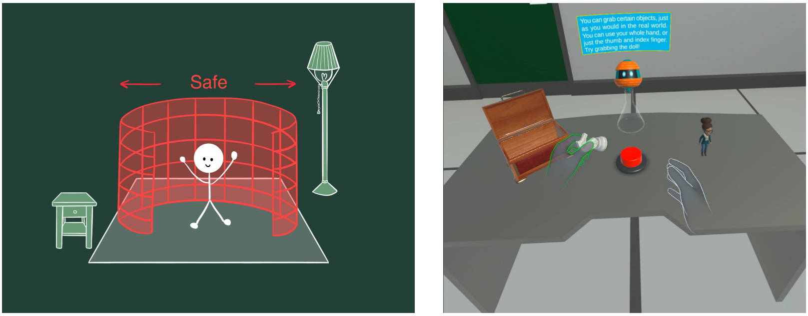


Figure 7.4: *VR activities should include a phase of familiarization with the space (left) as well as the interaction (right), independently of the subsequent learning task. The tutorial presented on the right picture has been conceptualized and implemented by Robin Hänni as part of a collaboration. The picture is used with authorization.*

In-VR embodied assessments are powerful to identify learning outcomes. Another major outcome of this work is novel evidence for the need to integrate in-VR embodied learning assessments when empirically evaluating learning outcomes. Previous work has already highlighted the importance of embodied assessments [Johnson-Glenberg, 2019; Tran et al., 2017].

When learners make sense of a novel concept, their understanding is expressed in gestures before it is expressed in speech or written form [Roth, 2001]. This means that if we evaluate embodied learning activities, especially short activities or activities focused on intuition building, with traditional written or spoken tests, we might not capture preliminary evidence for learning.

Moreover, evidence shows that cognitively relevant gestures used during a learning task only impacted learning outcomes when these students actually used these gestures during the assessment [Walkington et al., 2022]. Again, this effect is not captured with traditional forms of assessment, and might even prevent students from performing such gestures by occupying their hands with pencils, computer mice, and other instruments.

In this work, we complete these recommendations by providing evidence specific to VR. In Chapter 6, we demonstrated that, as opposed to tablets, VR did not suffer a negative effects of mathematics anxiety on interaction, and, as a consequence, learning outcomes. We believe that this result is due to the immersive properties of VR as it removes external social cues, and therefore the negative effect of social comparison for students with math anxiety. Out-of-VR forms of assessment might impact math anxious students negatively by bringing social cues back too soon in the learning process.

In conclusion, in-VR embodied learning assessments should be considered in embodied research, and, we believe, provide a fruitful research direction supporting the identification of embodied learning effects that are not captures with traditional forms of assessment.

7.2 Future Work

In the continuity of this work, we are interested in several research directions. In this section, we describe these directions and, when relevant, preliminary results of our own.

7.2.1 Further exploration of the design space

In this thesis, we explored the different elements of our design framework individually. However, future work should explore the relationships between these elements and the implications for design. For example, how can Digital Gloves (***DigiGlo***) be beneficial in the context embodied concreteness? Or how do learners influence interaction meaning?

7.2.2 Embodied assessments in Virtual Reality

As mentioned before, in Chapter 6, we identified the importance and need for in-VR embodied learning assessments. There are many benefits of such assessments such as capturing preliminary evidence of learning, identifying moderating effects of embodied gestures, and reduce the effect of mathematics anxiety on learning outcomes.

More generally, we believe that we do not currently have all the instruments to evaluate the learning outcomes of embodied learning activities. Future work should use hand-tracking algorithms and machine learning approaches to offer automatic embodied assessments in VR accounting for the diversity of learners' bodies [Spiel, 2021], and building on previous literature on in-VR questionnaires [Safikhani et al., 2021].

7.2.3 Automatic embodied interactive examples

In our work, we designed and implemented the different learning activities manually. However, technology offers another important advantage: the possibility to generate such activities automatically, and adjusted to the learners' progress.

For example, previous work described "Penrose", a system to automatically generate visualizations based on mathematical expressions [Ye et al., 2020]. In future work, we suggest to generate embodied and interactive representations of formal expressions, using embodied interaction as explored in our work. Following this direction, we implemented a first prototype of the Penrose system in Unity (Figure 7.5). In the future, we plan on integrating this implementation within a VR framework and extend it with embodied interaction.

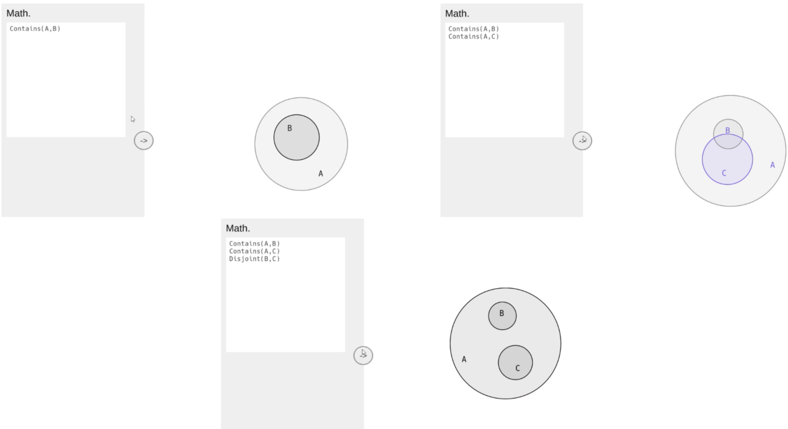


Figure 7.5: *Three screenshots of Amatheur, our implementation of the Penrose system [Ye et al., 2020], in Unity, C#. In the first screenshot, the learner creates an example of $A \supset B$. In the second screenshot, they add the rule $A \supset C$. In the last screenshot, they add $B \cap C = \emptyset$. The screenshots are cropped for clarity.*

7.2.4 Embodied formalisms

In Chapter 4, we identified the need to reconnect embodied learning activities to more formal instruction. In Chapter 5, we highlighted the potential of embodied concreteness to ground more abstract mathematics. However, we could and should go even further and investigate the connection between embodied learning activities and abstract symbols and formalisms.

To do so, one solution is to bring such symbols into the VR experience (Figure 7.6). This requires novel research on mathematics input in VR, that we investigated during this doctorate [Sansonetti et al., 2021].

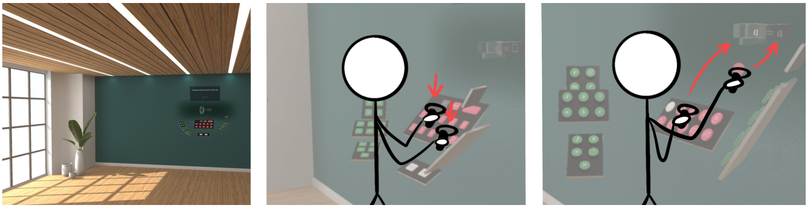


Figure 7.6: *Example of mathematics input in* VR *from our published research [Sansonetti et al., 2021].*

Another approach would be to create situations where students feel the need to create such symbols themselves, as presented in the multi-player scenario above [Games, 2015]. For example, in the context of linear algebra learning, we could imagine one situation where a student has a geometrical representation of a 2D space, and another player has access to an interactive tool to modify a normalized 2D space (Figure 7.7). Both players do not have directly access to each other's information and need to communicate to reach the space represented on the picture using the interactive tool. We believe that, after some time, the students will converge towards an efficient way of communicating such transformations, potentially close to the representations used in mathematics. In such situations, symbols used for communicating mathematical objects need not be characters, but rather hand gestures and movements or words and sounds.

7.2.5 Taxonomy of concreteness

In Chapter 5, we conceptualized embodied as a form of concreteness, and importantly, presented evidence of a verbal dispute in the field of concreteness for mathematics education [Chalmers, 2011]. We argue that scholars

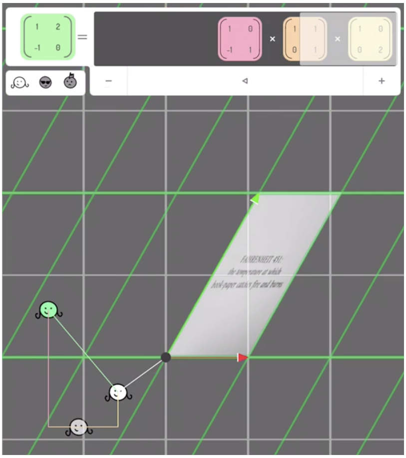


Figure 7.7: *Our first prototype for a tool to understand and communicate linear algebra through embodied interaction. Learners can interact with the space using either the matrices directly, or the vectors of the space. Moreover, they can add up to three "spies" in the space to observe their movement as they play the transformation by pressing the ◁ button. At the beginning of the activity, however, we envision a scenario where learners do not have access to the matrix representation yet, and have to communicate transformations in their own way, naturally progressing towards a more efficient representation. Finally, although the current prototype is implemented on tablet, the same concept can be explored in* VR.

and educators often use a wide range of meanings for the words "concrete" and "abstract". This has two impacts: (1) It makes the identification of the role of concreteness for mathematics learning difficult as the specific meanings are rarely rigorously defined, (2) It results in diverging empirical results that could be better understood within a multi-dimensional space accounting for the diversity of meanings.

Indeed, concreteness and abstraction are often considered as polar opposites organized along a one dimensional spectrum. In our work, we provided arguments against this representation [Chatain et al., 2023c]. Since then, we started looking into this issue more generally and used a GloVe model to identify the spatial organization of different meanings of "concrete" and "abstract" over a corpus of papers investigating concreteness in mathematics education [Pennington et al., 2014]. Our preliminary results, illustrated on Figure 7.8, show that the space is at least 3-dimensional.

In future work, we offer to pursue this research and work on a taxonomy of “concreteness” and “abstraction” that could be used as a tool to support rigorous definitions in empirical research, as well as a framework for meta-analyses across the field.

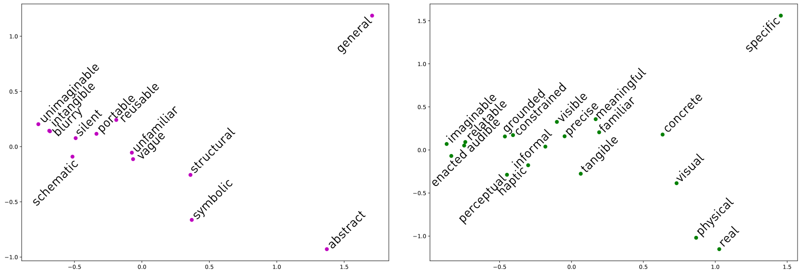


Figure 7.8: *Example of a preliminary exploration of the meanings of “concrete” and “abstract”, conducted in collaboration with Charlotte Müller and Keny Chatain. We obtained these figures by training a GloVe model over a corpus of research papers focused on concreteness in mathematics education [Pennington et al., 2014]. As defined by a Principal Component Analysis, the spaces of concreteness and abstraction are at least 3-dimensional. For clarity, we display the meanings in a 2D space defined by the two main components identified.*

We believe that the importance of this work can extend over other fields as well. For example, a same issue has been identified in chemistry, where scale often plays an important role as an element can be concrete as it exists in the physical world, yet abstract as it is too small to be perceived with human eyes [Müller et al., 2023].

7.3 Conclusion

In this work, we explored how to design embodied interaction to support embodied sense-making in VR. Specifically, we addressed three challenges: the lack of empirical studies in this field, especially in higher education, and the lack of theory-informed design guidelines for interaction in this context.

First, we described the framework we operate in and argued that three forms of embodiment should be considered: embodied cognition, embodied interaction, and avatar embodiment. We offered a system representation of each of these concepts and an overall representation of the whole framework. We then presented the affordances of VR to support embodied learning of mathematics.

To answer our research question, we focused on the different levels of relevance identified: avatar, interaction, context, learner and bodily actions.

At the avatar level, we presented "Digital Gloves", a novel interaction mechanism to support embodied interaction and reduce split-attention effect by co-locating input and display.

At the interaction level, we offered an empirical evaluation of the impact of the degree and type of embodiment on usability and learning. Our results show that not all embodied interaction supports embodied cognition, and offer design recommendation for future work.

At the context level, we conceptualized embodiment as a form of concreteness and demonstrated that embodied concreteness is a powerful tool to ground abstract mathematics without impairing transfer.

We then focused on learners and bodily actions and offered design recommendations for embodied learning activity accounting for individual differences and informed by research on both directed and spontaneous bodily actions.

Finally, we concluded by listing take-away messages aggregated over the entirety of our work as well as relevant avenues for future work.

In conclusion, our works demonstrates that embodied learning activities in VR, if designed properly, can benefit mathematics education, for example by grounding in concreteness and offering a safe space for math anxious individuals. However, certain tools should still be developed to identify the complete extent of these effects, such as automatic in-VR embodied learning assessments.

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This appendix includes:

1. Candidate's Resume
2. Questionnaires for the quantitative study presented in Chapter 4, as well as the quantitative study presented in Chapter 6
    - Preparatory intervention questionnaire
    - PS-I intervention: Pre-intervention questionnaire
    - PS-I intervention: Mid-intervention questionnaire for participants on tablets
    - PS-I intervention: Mid-intervention questionnaire for participants in VR
    - PS-I intervention: Post-intervention questionnaire
3. Learning test for the concreteness study presented in Chapter 5

JULIA CHATAIN

Researcher in Computer Science and Learning Sciences

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EXPERIENCE

Doctoral Student

ETH Zurich

July 2019 – March 2023 | Zurich, Switzerland

- Supervised by Prof. Dr. Robert W. Sumner (Game Technology Center) and Prof. Dr. Manu Kapur (Professorship for Learning Sciences and Higher Education)
- Exploring the role of concreteness in learning mathematics, and in particular how to ground abstract mathematics with embodied interaction in Virtual Reality
- Head Teaching Assistant for "Computer Science I" and "Data structures and Algorithms" courses

Research Engineer

Game Technology Center, ETH Zurich

March 2017 – June 2019 | Zurich, Switzerland

- Developed a visual programming tool to create video games on mobile, presented at the World Economic Forum 2019
- Co-developed a playful Augmented Reality Christmas catalog for Franz Carl Weber
- Co-developed "Gnome Trader", an Augmented Reality trading game for Smart Cities (European Project)

Research Student

Potioc, Inria Bordeaux Sud-Ouest

Sept 2015 – Jan 2017 | Bordeaux, France

- Developed Symapse, a Spatial Augmented Reality tool to let citizens share, through drawings and scribbles, their impression of a city
- Co-developed FlyMap, a Spatial Augmented Reality drone to guide students through the campus of Stanford University by projecting an interactive map on the floor

Software Engineer Intern

Google

July 2016 – Sept 2016 | Zurich, Switzerland

- Developed an evaluation tool for a reverse geocoding algorithm

Program Manager Intern

Microsoft

August 2012 | Paris, France

- Implemented animations moving on the rhythm of the user's music.

RESEARCH INTERESTS

Human Learning | XR | Human Computer Interaction | Embodied Interaction | Embodied Cognition | Abstraction | Mathematics Education

SKILLS

Unity | C# | C++ | Java | Python | R | Photoshop | Illustrator | InDesign | Student supervision | Team management

EDUCATION

Master of Science in Computer Science

École Polytechnique Fédérale de Lausanne

2013 – 2015 | Lausanne, Switzerland

Engineering Degree in Computer Science, completed with "Outstanding Investment"

École polytechnique

2010 – 2013 | Paris, France

Preparation to "Grandes Écoles", Elite class (MP*), focused on mathematics

Lycée Descartes

2008 – 2010 | Tours, France

LANGUAGES

French ●●●●●
English ●●●●●
German ●○○○○

HOBBIES

Traveling | Crafting | Reading | Brazilian Jiu-Jitsu | Video Games | Board Games | Blogging

OUTREACH, SERVICE, AND VOLUNTEERING

- Communications Committee member at the Special Interest Group on Computer–Human Interaction (SIGCHI '23).
- Web Chair at the Annual Symposium on Computer-Human Interaction in Play (CHI PLAY '21)
- Student Volunteer at the Annual Symposium on Computer-Human Interaction in Play (CHI PLAY '20)
- Promoting Computer Science and Mathematics to young girls and women: Girls Code Too Switzerland, Coding Club des Filles EPFL, Schnupperstudium ETH, Django Girls Bordeaux and Lausanne, etc
- Organizing Game Events for the ETH Game Technology Center and Disney Research Zurich

TRANSFER AND ENTREPRENEURSHIP

- Supervisory board member for Girls Code Too (2021-2022)
- Learning Sciences consultant for Hackworth Ltd (2020-2021)
- Scientific advisor for Enlightware GmbH (2018-2021)
- Computer graphics intern for Fitle (2014)

SUPERVISION

- Martina Kessler - Master Thesis - Feb. 2023 - Playful Experiences with Embodied Interaction in Augmented Reality
- Robin Hänni - Bachelor Thesis - Dec. 2022 - Virtual Reality Cytology Lab for Risk Awareness
- Dominic Weibel - Master Thesis - Sept. 2022 - Co-Designing a Computer Science Learning Game for Girls with Girls
- Bibin Muttappillil - Bachelor Thesis - March 2022 - Design and Evaluation of Embodied Interaction in Virtual Reality for Learning Derivatives
- Bodo Brägger - Master Thesis - Feb. 2022 - Gender Equality in Computer Science: Video Games as Preparation for Future Learning
- Rudolf Varga - Master Thesis - Sept. 2021 - Learning Graph Theory with Embodied Interaction in Virtual Reality
- Luigi Sansonetti - Master Thesis - May 2021 - Mathematics Input for Educational Applications in Virtual Reality
- Virginia Ramp - Master Thesis - Oct. 2020 - Embodied Analysis in Virtual Reality
- Lea Reichardt - Bachelor Thesis - Dec. 2019 - VR Game Prototype for Hand Tracking and Projection

SELECTED PUBLICATIONS

- **Chatain**, Julia, Rudolf Varga, Violaine Fayolle, Manu Kapur, Robert W. Sumner. "Grounding Graph Theory in Embodied Concreteness with Virtual Reality". In Proceedings of the Seventeenth International Conference on Tangible, Embedded, and Embodied Interaction. (2023)
- **Chatain**, Julia, Virginia Ramp, Venera Gashaj, Violaine Fayolle, Manu Kapur, Robert W. Sumner, Stéphane Magnenat. "Grasping Derivatives: Teaching Mathematics through Embodied Interactions using Tablets and Virtual Reality". In Interaction Design and Children (IDC'22). (2022)
- Sansonetti, Luigi, Julia **Chatain**, Pedro Caldeira, Violaine Fayolle, Manu Kapur, Robert W. Sumner. "Mathematics Input for Educational Applications in Virtual Reality". In International Conference on Artificial Reality and Telexistence and Eurographics Symposium on Virtual Environments (ICAT-EGVE). (2021)
- **Chatain**, Julia, Danielle M. Sisserman, Lea Reichardt, Violaine Fayolle, Manu Kapur, Robert W. Sumner, Fabio Zünd, Amit H. Bermano. "DigiGlo: Exploring the Palm as an Input and Display Mechanism through Digital Gloves." In Proceedings of the Annual Symposium on Computer-Human Interaction in Play (CHI PLAY '20), November 2–4, 2020, Virtual Event, Canada. ACM, New York, NY, USA, 12 pages. (2020)
- **Chatain**, Julia, Olivier Bitter, Violaine Fayolle, Robert W. Sumner, and Stéphane Magnenat. "A Creative Game Design and Programming App." In Motion, Interaction and Games, pp. 1-6. (2019)

SELECTED TALKS

- "Grounding Abstract Mathematics with Embodied Interaction". Saarland University, Germany, 2022.
- Panelist "Children & computing: increasing gender diversity". Interaction Design and Children (IDC) 2022.
- "Grasping Mathematics with Embodied Interaction in Virtual Reality: The Case of Derivatives". Future Learning Initiative Colloquium, 2022.
- Panelist "IDC for Gender Balance: How can we engage more girls in informatics?". Interaction Design and Children (IDC) 2021.
- "Grasping Mathematics in Virtual Reality". Future Learning Initiative Colloquium, 2021.

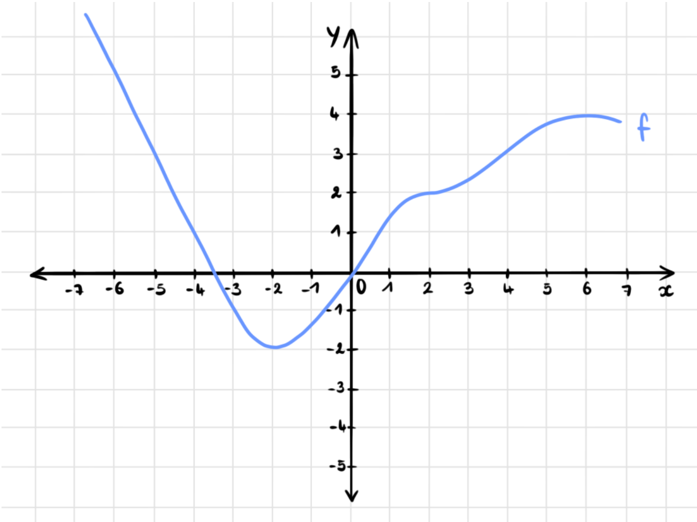
Questionnaire 1.1

Participant Id = ___________________

Please fill in the following questionnaire. This questionnaire is not part of your curriculum and will not influence your grade in any way. If you do not know how to solve a question, you can simply skip it.

Section 1.

Please, answer the questions in this section using the following graph.



a. What is the value of f(6)?

f(6) = ___________

b. What is the value of f(-2)?

f(-2) = ___________

c. What is the value of f(-5)?

f(-5) = ___________

d. On the interval]-6,-2[, f is:

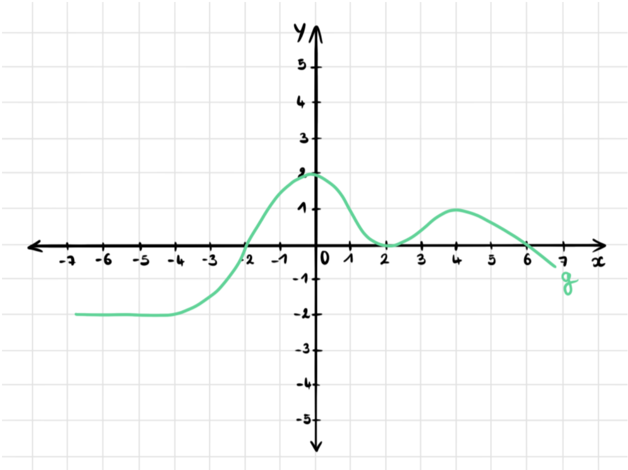
◯ Increasing
◯ Decreasing
◯ Constant
◯ None of the above

e. On the interval]2,6[, f is:

◯ Increasing
◯ Decreasing
◯ Constant
◯ None of the above

Section 2.

Please, answer the questions in this section using the following graph. All these questions focus on the interval [-6,6].



a. Select all the values where g is null (that is, g(x) = 0).

☐ -6 ☐ -5 ☐ -4 ☐ -3 ☐ -2 ☐ -1 ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

b. On which interval(s) is g positive?

____

c. On which interval(s) is g negative?

____

Section 3.

For this section, please use the graph of the function g above.

a. What is the maximum value of g?

☐ -5 ☐ -4 ☐ -3 ☐ -2 ☐ -1 ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

b. What is the minimum value of g?

☐ -5 ☐ -4 ☐ -3 ☐ -2 ☐ -1 ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

c. Does g have any local maximum?

◯ Yes ◯ No

If yes, which one(s)? Select all the values of x where g(x) is a local maximum.

☐ -7 ☐ -6 ☐ -5 ☐ -4 ☐ -3 ☐ -2 ☐ -1 ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

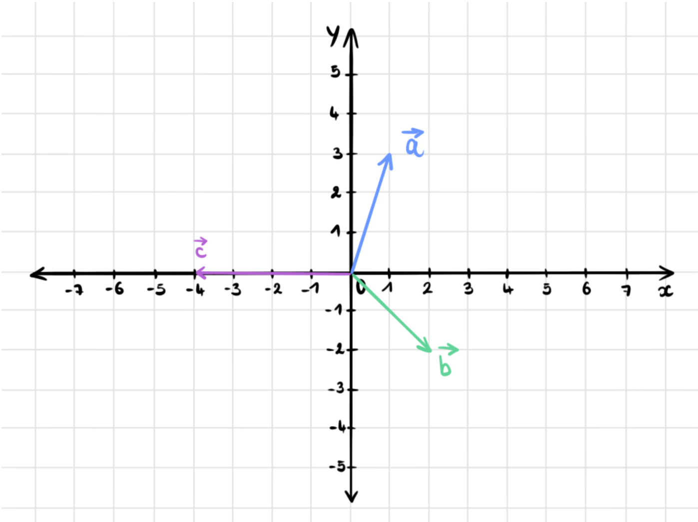
d. Does g have any local minimum?

◯ Yes ◯ No

If yes, which one(s)? Select all the values of x where g(x) is a local minimum.

☐ -7 ☐ -6 ☐ -5 ☐ -4 ☐ -3 ☐ -2 ☐ -1 ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

Section 4.



a. In the above picture, what are the coordinates of vector a?
The coordinates of a are (_____,_____)

b. In the above picture, what are the coordinates of vector b?
The coordinates of b are (_____,_____)

c. In the above picture, what are the coordinates of vector c?
The coordinates of c are (_____,_____)

Questionnaire 1.2

Participant Id = __________________

Please fill in all the questions of the following questionnaire.

Gender

◯ Female ◯ Male ◯ Other ◯ Prefer not to say

Age

__________ years old

Height

_________ cm

Mother Tongue

____

Occupation (if you are a student, please indicate your grade, school, and program)

____

Are you a repeating student?

◯ Yes ◯ No

What is your math grade?

_________/6

I play video games (console, desktop, or mobile)

◯ Every day
◯ Several times a week
◯ Once a week
◯ Once a month
◯ Less

I use a VR headset (Oculus, Vive, etc)

◯ Every day
◯ Several times a week
◯ Once a week
◯ Once a month
◯ Less

I use a tablet (iPad, Galaxy tab, etc)

◯ Every day
◯ Several times a week
◯ Once a week
◯ Once a month
◯ Less

I use movable controllers (like Wii remote controllers, VR controllers, etc)
◯ Every day
◯ Several times a week
◯ Once a week
◯ Once a month
◯ Less

I use a hand tracking system (Ultraleap, Leap motion, Oculus Quest, etc)
◯ Every day
◯ Several times a week
◯ Once a week
◯ Once a month
◯ Less

Name your three favorite hobbies (number 1 is your favorite)

1. ______________________

2. ______________________

3. ______________________

What is your dominant hand?
◯ Right ◯ Left ◯ Both

Do you have to wear glasses or contact lenses?

☐ Glasses

☐ Contact lenses

☐ Nothing

If you wear glasses, will you keep your glasses during the Virtual Reality experience?
◯ Yes ◯ No

Questionnaire 1.3

Participant Id = ___________________

Please fill in all the questions of the following questionnaire. For each of the statements, indicate how much it applies to you from "Not true at all about me" (1) to "Very true about me" (7) by checking the according box.

I notice differences in the way my body reacts to various foods.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I can always tell when I bump myself whether or not it will become a bruise.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I always know when I've exerted myself to the point where I'll be sore the next day.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I am always aware of changes in my energy level when I eat certain foods.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I know in advance when I'm getting the flu.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I know I'm running a fever without taking my temperature.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I can distinguish between tiredness because of hunger and tiredness because of lack of sleep.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I can accurately predict what time of day lack of sleep will catch up with me.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I am aware of a cycle in my activity level throughout the day.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I DON'T notice seasonal rhythms and cycles in the way my body functions.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

As soon as I wake up in the morning I know how much energy I'll have during the day.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I can tell when I go to bed how well I will sleep that night.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I notice distinct body reactions when I'm fatigued.

Not true at all about me ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ → Very true about me

I notice specific body responses to changes in the weather.

Not true at all about me	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐	6 ☐	7 ☐	Very true about me

I can predict how much sleep I will need at night in order to wake up refreshed.

Not true at all about me	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐	6 ☐	7 ☐	Very true about me

When my exercise habits change, I can predict very accurately how that will affect my energy level.

Not true at all about me	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐	6 ☐	7 ☐	Very true about me

There seems to be a "best" time for me to go to sleep at night.

Not true at all about me	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐	6 ☐	7 ☐	Very true about me

I notice specific bodily reactions to being overhungry.

Not true at all about me	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐	6 ☐	7 ☐	Very true about me

Questionnaire 1.4

Participant Id = ___________________

Please fill in all the questions of the following questionnaire. Please indicate how anxious these situations make you feel, from “Low anxiety” (1) to “High anxiety” (5).

Having to use the tables in the back of a math book

Low anxiety ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ → High anxiety

Thinking about an upcoming math test 1 day before

Low anxiety ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ → High anxiety

Watching a teacher work an algebraic equation on the blackboard

Low anxiety ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ → High anxiety

Taking an examination in a math course

Low anxiety ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ → High anxiety

Being given a homework assignment of many difficult problem that is due the next class meeting

Low anxiety ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ → High anxiety

Listening to a lecture in math class

Low anxiety ← 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ → High anxiety

Listening to another student explain a math formula

Low anxiety 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ High anxiety

Being given a "pop" quiz in math class

Low anxiety 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ High anxiety

Starting a new chapter in a math book

Low anxiety 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ High anxiety

References

Shields, S. A., Mallory, M. E., & Simon, A. (1989). The body awareness questionnaire: reliability and validity. Journal of personality Assessment, 53(4), 802-815.

Hopko, D. R., Mahadevan, R., Bare, R. L., & Hunt, M. K. (2003). The abbreviated math anxiety scale (AMAS) construction, validity, and reliability. Assessment, 10(2), 178-182.

Questionnaire 2.0 (pre)

Participant Id = ____________________

Please fill in the following questionnaire according to how you feel.

General Discomfort

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Fatigue

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Headache

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Eyestrain

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Difficulty Focusing

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Increased salivation

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Sweating

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Nausea

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Difficulty concentrating

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Fullness of head

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Blurred vision

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Dizzy (eyes open)

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Dizzy (eyes closed)

None ☐ 0 ☐ 1 ☐ 2 ☐ 3 Severe

Vertigo

None 0 1 2 3 Severe

Stomach awareness

None 0 1 2 3 Severe

Burping

None 0 1 2 3 Severe

Reference

Kennedy, R. S., Lane, N. E., Berbaum, K. S., & Lilienthal, M. G. (1993). Simulator sickness questionnaire: An enhanced method for quantifying simulator sickness. The international journal of aviation psychology, 3(3), 203-220.

Questionnaire 2.1 (mid)

Participant Id = ____________________

Please fill in the following questionnaire according to how you feel.

General Discomfort

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Fatigue

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Headache

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Eyestrain

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Difficulty Focusing

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Increased salivation

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Sweating

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Nausea

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Difficulty concentrating

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Fullness of head

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Blurred vision

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Dizzy (eyes open)

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Dizzy (eyes closed)

None 0 ☐ 1 ☐ 2 ☐ 3 ☐ Severe

Vertigo

None ← ☐0 ☐1 ☐2 ☐3 → Severe

Stomach awareness

None ← ☐0 ☐1 ☐2 ☐3 → Severe

Burping

None ← ☐0 ☐1 ☐2 ☐3 → Severe

Questionnaire 2.2 (mid)

Participant Id = ___________________

Please fill in the following questionnaire. For each of the statements, indicate how strongly you agree with it, from “Strongly disagree” (1) to “Strongly agree” (5). All the statements apply to **math class**.

I think that I would like to use this system frequently

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 Strongly agree

I found the system unnecessarily complex

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 Strongly agree

I found the system was easy to use

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 Strongly agree

I think that I would need the support of a technical person to be able to use this system

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 Strongly agree

I found the various functions in this system were well integrated

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 Strongly agree

I thought there was too much inconsistency in this system

Strongly disagree — 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ — Strongly agree

I would imagine that most people would learn to use this system very quickly

Strongly disagree — 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ — Strongly agree

I found the system very cumbersome to use

Strongly disagree — 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ — Strongly agree

I felt very confident using the system

Strongly disagree — 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ — Strongly agree

I needed to learn a lot of things before I could get going with this system

Strongly disagree — 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ — Strongly agree

Questionnaire 2.3 (mid)

Participant Id = ___________________

Please read each statement and answer on a 1 to 7 scale indicating how much each statement applied to you during the experiment. There are no right or wrong answers. Please answer spontaneously and intuitively.

It felt like I could control the curve as if I was directly manipulating in the real world

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

The movements of the curve were caused by my movements

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt as if the movements of the curve were influencing my own movements

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt as if the curve was moving by itself

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

Questionnaire 2.4 (mid)

Participant Id = ___________________

Please read each statement and answer on a 1 to 5 scale indicating how much each statement applied to you during the experiment. There are no right or wrong answers. Please answer spontaneously and intuitively.

I used the score

Never ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 Very often

References

Kennedy, R. S., Lane, N. E., Berbaum, K. S., & Lilienthal, M. G. (1993). Simulator sickness questionnaire: An enhanced method for quantifying simulator sickness. The international journal of aviation psychology, 3(3), 203-220.

Brooke, J. (1996). Sus: a “quick and dirty’usability. Usability evaluation in industry, 189(3).

Gonzalez-Franco, M., & Peck, T. C. (2018). Avatar embodiment. towards a standardized questionnaire. Frontiers in Robotics and AI, 5, 74.

Questionnaire 2.1 (mid)

Participant Id = ____________________

Please fill in the following questionnaire according to how you feel.

General Discomfort

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Fatigue

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Headache

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Eyestrain

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Difficulty Focusing

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Increased salivation

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Sweating

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Nausea

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Difficulty concentrating

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Fullness of head

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Blurred vision

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Dizzy (eyes open)

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Dizzy (eyes closed)

None	0	1	2	3	Severe
	☐	☐	☐	☐	

Vertigo

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Stomach awareness

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Burping

None	0 ☐	1 ☐	2 ☐	3 ☐	Severe

Questionnaire 2.2 (mid)

Participant Id = ___________________

Please fill in the following questionnaire. For each of the statements, indicate how strongly you agree with it, from “Strongly disagree” (1) to “Strongly agree” (5). All the statements apply to **math class**.

I think that I would like to use this system frequently

	1	2	3	4	5	
Strongly disagree	☐	☐	☐	☐	☐	Strongly agree

I found the system unnecessarily complex

	1	2	3	4	5	
Strongly disagree	☐	☐	☐	☐	☐	Strongly agree

I found the system was easy to use

	1	2	3	4	5	
Strongly disagree	☐	☐	☐	☐	☐	Strongly agree

I think that I would need the support of a technical person to be able to use this system

	1	2	3	4	5	
Strongly disagree	☐	☐	☐	☐	☐	Strongly agree

I found the various functions in this system were well integrated

	1	2	3	4	5	
Strongly disagree	☐	☐	☐	☐	☐	Strongly agree

I thought there was too much inconsistency in this system

Strongly disagree — 1 □ 2 □ 3 □ 4 □ 5 □ — Strongly agree

I would imagine that most people would learn to use this system very quickly

Strongly disagree — 1 □ 2 □ 3 □ 4 □ 5 □ — Strongly agree

I found the system very cumbersome to use

Strongly disagree — 1 □ 2 □ 3 □ 4 □ 5 □ — Strongly agree

I felt very confident using the system

Strongly disagree — 1 □ 2 □ 3 □ 4 □ 5 □ — Strongly agree

I needed to learn a lot of things before I could get going with this system

Strongly disagree — 1 □ 2 □ 3 □ 4 □ 5 □ — Strongly agree

Questionnaire 2.3.1 (mid)

Participant Id = ___________________

Please read each statement and answer on a 1 to 7 scale indicating how much each statement applied to you during the experiment. There are no right or wrong answers. Please answer spontaneously and intuitively.

I felt like the virtual hands were my virtual hands.

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt like the virtual fingers were my fingers.

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

The virtual hands felt like human hands.

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

It felt like the virtual hands belonged to me.

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

The movements of the virtual hands felt like they were my movements.

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt like I was controlling the movements of the virtual hands.

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt like I was causing the movements of the virtual hands.

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 Strongly agree

The movements of the virtual hands were in sync with my own movements.

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 Strongly agree

I felt like the form or appearance of my own hands changed.

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 Strongly agree

I felt like the weight of my own hands had changed.

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 Strongly agree

I felt like the length of my own hands had changed.

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 Strongly agree

I felt like the width of my own hands had changed.

Strongly disagree ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 Strongly agree

Questionnaire 2.3.2 (mid)

Participant Id = ___________________

Please read each statement and answer on a 1 to 7 scale indicating how much each statement applied to you during the experiment. There are no right or wrong answers. Please answer spontaneously and intuitively.

It felt like I could control the curve as if I was directly manipulating in the real world

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

The movements of the curve were caused by my movements

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt as if the movements of the curve were influencing my own movements

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

I felt as if the curve was moving by itself

	1	2	3	4	5	6	7	
Strongly disagree	☐	☐	☐	☐	☐	☐	☐	Strongly agree

Questionnaire 2.4 (mid)

Participant Id = ___________________

Please read each statement and answer on a 1 to 5 scale indicating how much each statement applied to you during the experiment. There are no right or wrong answers. Please answer spontaneously and intuitively.

I used the mini-display

	1	2	3	4	5	
Never	☐	☐	☐	☐	☐	Very often

I used the score

	1	2	3	4	5	
Never	☐	☐	☐	☐	☐	Very often

References

Kennedy, R. S., Lane, N. E., Berbaum, K. S., & Lilienthal, M. G. (1993). Simulator sickness questionnaire: An enhanced method for quantifying simulator sickness. The international journal of aviation psychology, 3(3), 203-220.

Brooke, J. (1996). Sus: a “quick and dirty’usability. Usability evaluation in industry, 189(3).

Roth, D., & Latoschik, M. E. (2019). Construction of a validated virtual embodiment questionnaire. arXiv preprint arXiv:1911.10176.

Gonzalez-Franco, M., & Peck, T. C. (2018). Avatar embodiment. towards a standardized questionnaire. Frontiers in Robotics and AI, 5, 74.

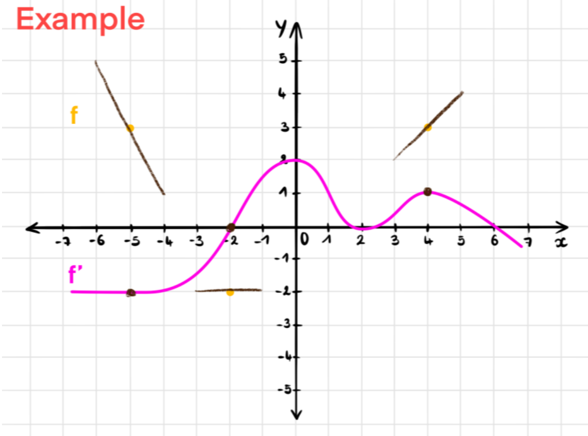
Questionnaire 2.4 (post)

Participant Id = ____________________

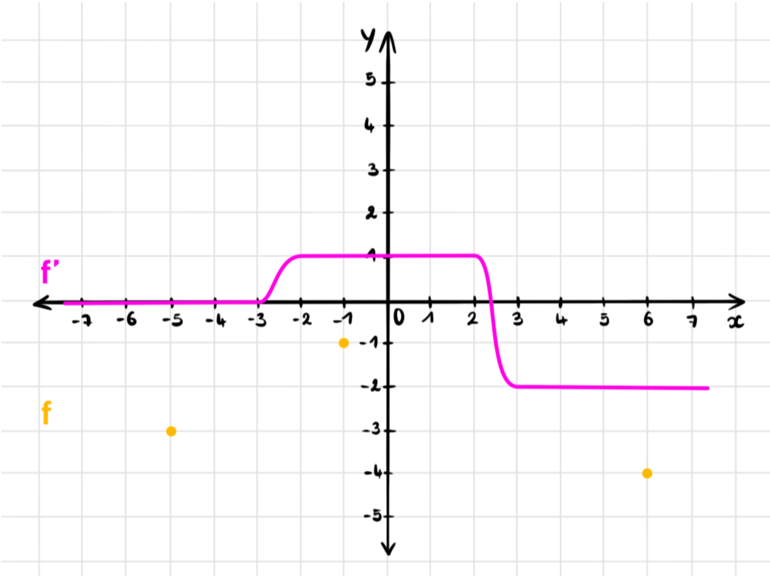
Please fill in the following questionnaire. This questionnaire is not part of your curriculum and will not influence your grade in any way.

Section 1.

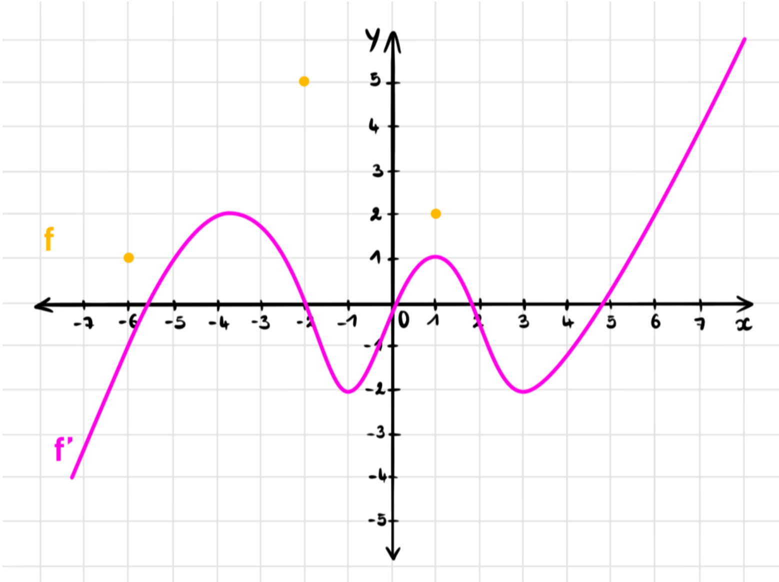
On the following graphs, the target **derivative f'** is represented in pink. Only a few points of the **function f** are visible. For each of these points, draw the desired tangent.



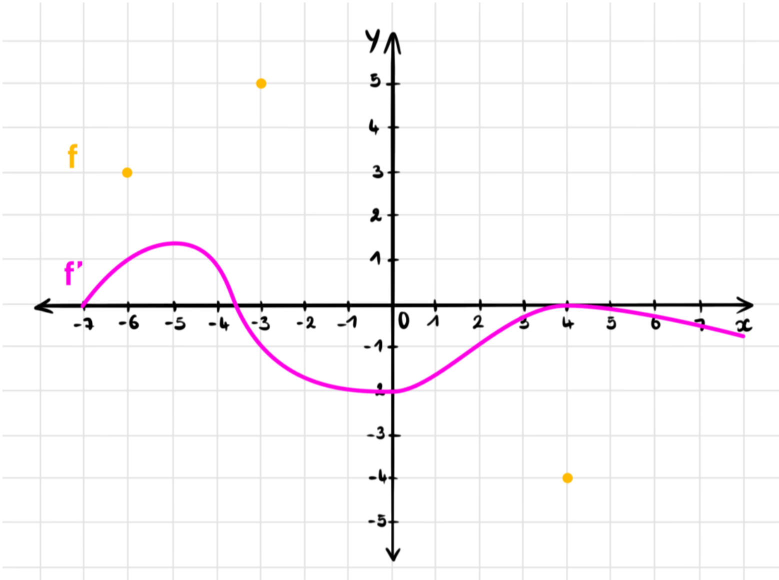
Task 1:



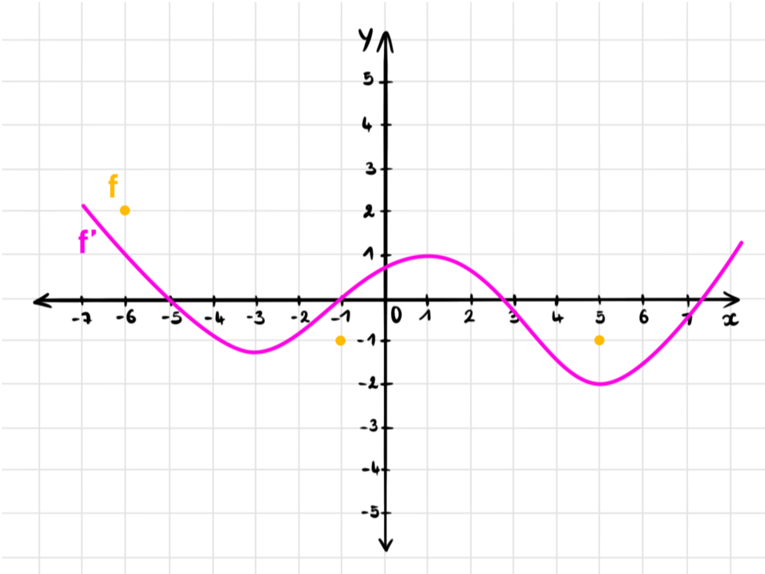
Task 2:


y
x
f
f'
5
4
3
2
1
-1
-2
-3
-4
-5
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7

Task 3:


y
x
f
f'
5
4
3
2
1
-1
-3
-4
-5
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7

Task 4:



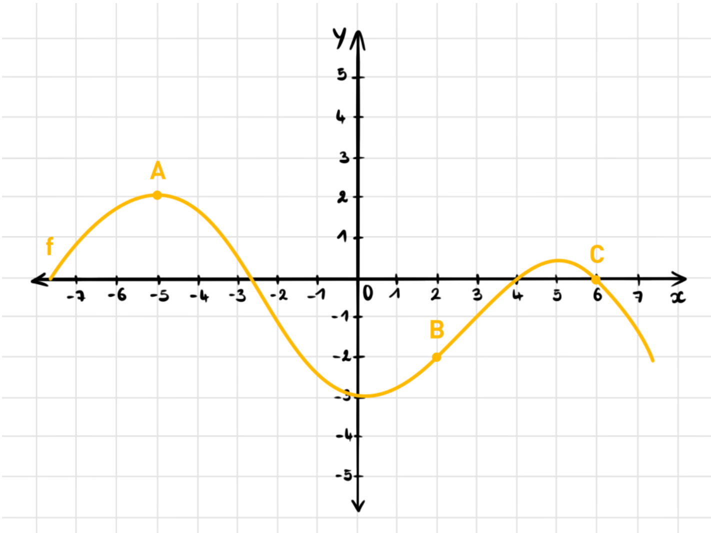
Questionnaire 2.5 (post)

Participant Id = ____________________

Please fill in the following questionnaire. This questionnaire is not part of your curriculum and will not influence your grade in any way.

Section 1.

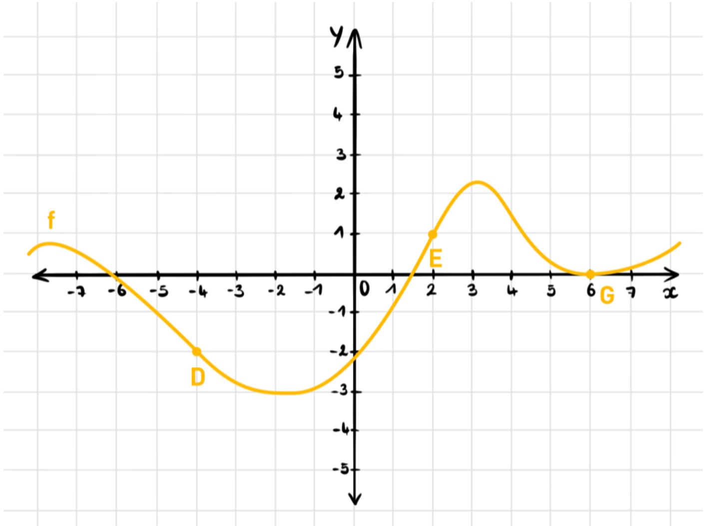
On the following graphs, the **function f** is represented in orange.



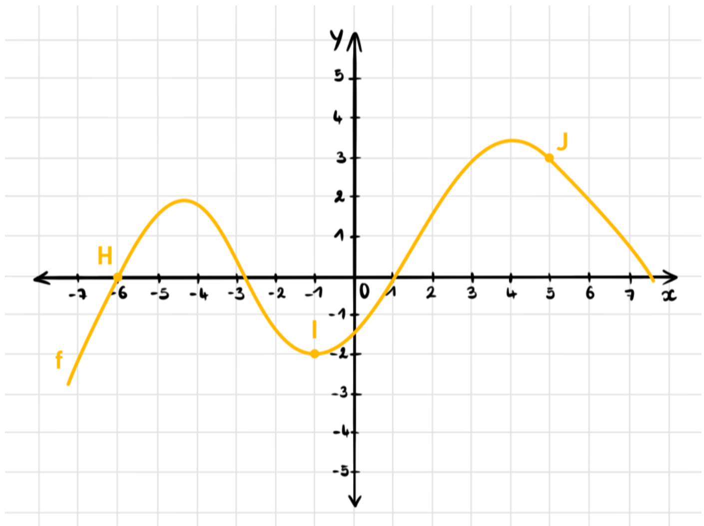
What is the value of the **derivative f'** at point A? f'(-5) = __________________

What is the value of the **derivative f'** at point B? f'(2) = __________________

What is the value of the **derivative f'** at point C? f'(6) = __________________



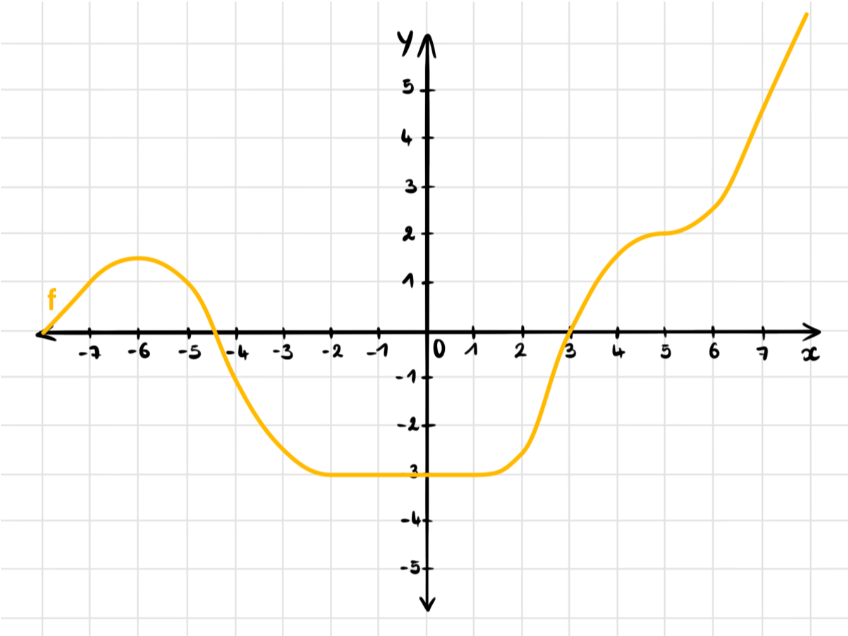
What is the value of the **derivative f'** at point D? f'(-4) = ___________________
What is the value of the **derivative f'** at point E? f'(2) = ___________________
What is the value of the **derivative f'** at point G? f'(6) = ___________________



What is the value of the **derivative f'** at point H? f'(-6) = ___________________
What is the value of the **derivative f'** at point I? f'(-1) = ___________________
What is the value of the **derivative f'** at point J? f'(5) = ___________________

Section 2.

On the following graph, the **function f** is represented in orange. Draw the **derivative f'** of the function on the graph.



Section 3.

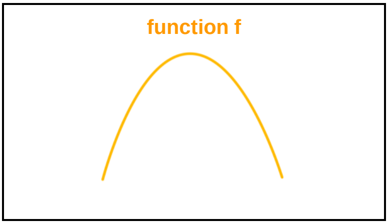
In this section, the left column shows the sign table of the **derivative f'**. A sign table shows the evolution of the sign of the derivative along the x axis. From example, $+0-$ means that the **derivative f'** is first positive over a certain interval, then crosses zero on one point, and becomes negative over an interval.

In each exercise, you have to look at the sign table of the **derivative f'** on the left, and draw a possible shape of the **function f** in the frame on the right.

Example:

derivative f'

$+ \quad 0 \quad -$ $\rightarrow$



derivative f'

$- \quad 0 \quad + \quad 0 \quad -$ $\rightarrow$

function f

derivative f'

$+ \quad 0 \quad - \quad 0 \quad +$ $\rightarrow$

function f

derivative f'

$+ \quad 0 \quad + \quad 0 \quad +$ $\rightarrow$

function f

derivative f’

$- \quad 0 \quad - \quad 0 \quad - \qquad \rightarrow$

function f

derivative f’

$+ \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \qquad \rightarrow$

function f

derivative f’

$- \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \qquad \rightarrow$

function f

derivative f’

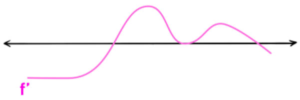
$+ \quad 0 \quad + \quad 0 \quad - \quad 0 \quad - \qquad \rightarrow$

function f

Section 4.

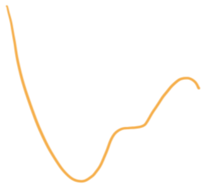
In this exercise, you have to look at the **derivative f'** on the left, and draw the shape of the **function f** in the frame on the right.

derivative f'

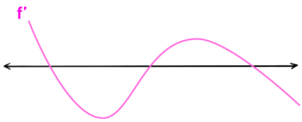


→

function f



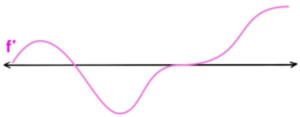
derivative f'



→

function f

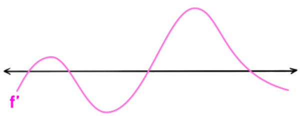
derivative f'



→

function f

derivative f'



→

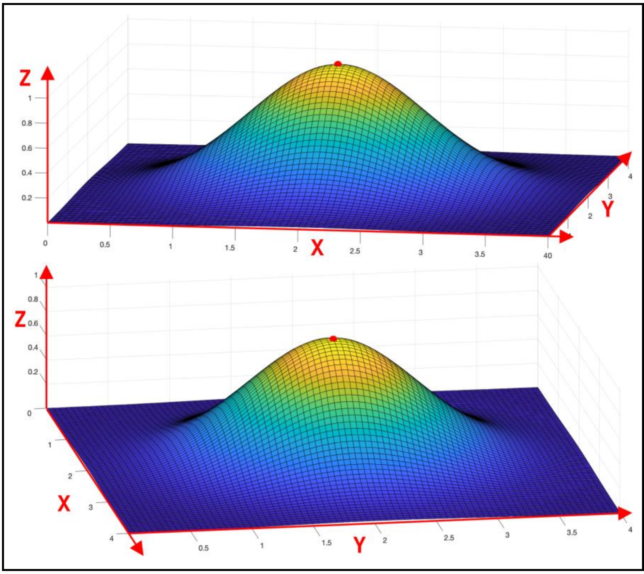
function f

Section 5.

In this section, all the exercises will be in 3D spaces. The **function** is represented on the left, from two different perspectives, and with a specific **point** indicated in red. On the right, you have to select the correct value of the **derivative** at this **point**.

In 3D, the **derivative** at a specific point has two components. The x component of the derivative vector represents the slope along the x axis: how much does the surface go up or down if we follow the x-axis in the direction of its arrow. Similarly, the y component represents the slope along the y axis.

Be mindful of the direction of the axes!



Derivative along x

○ -1

○ 0

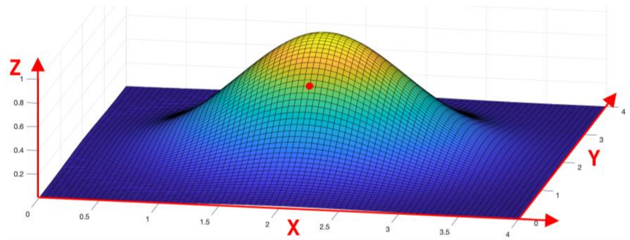
○ 1

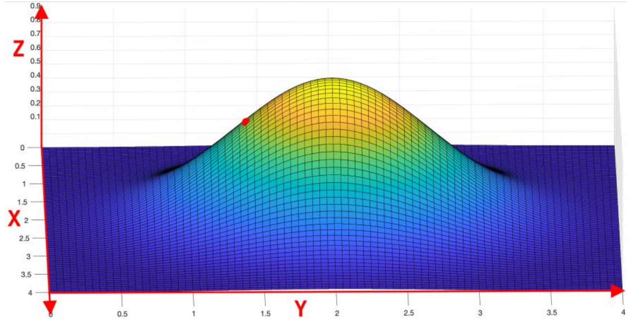
Derivative along y

○ -1

○ 0

○ 1



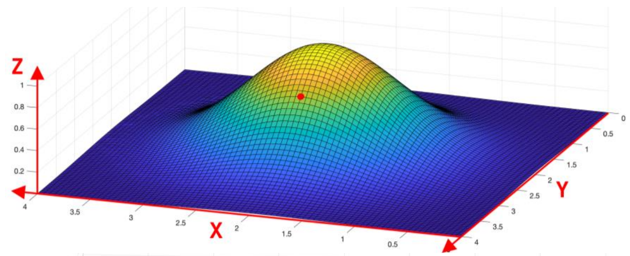


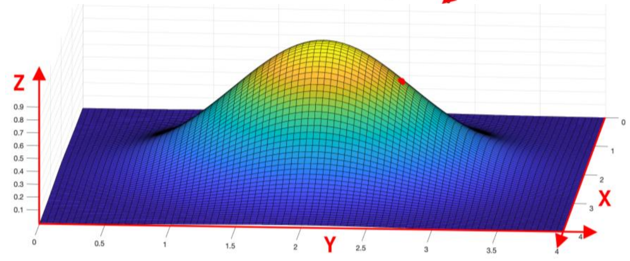
Derivative along x

- ◯ -0.8
- ◯ 0
- ◯ 0.8

Derivative along y

- ◯ -0.8
- ◯ 0
- ◯ 0.8





Derivative along x

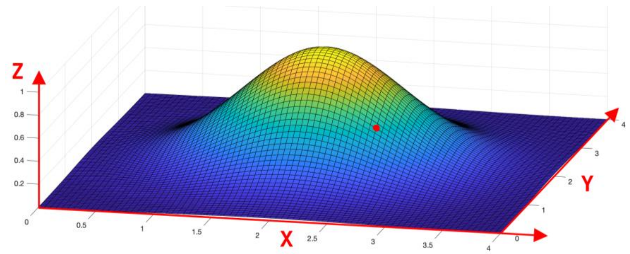
- ◯ -0.8
- ◯ 0
- ◯ 0.8

Reverted axis

Derivative along y

- ◯ -0.8
- ◯ 0
- ◯ 0.8

Derivative along x

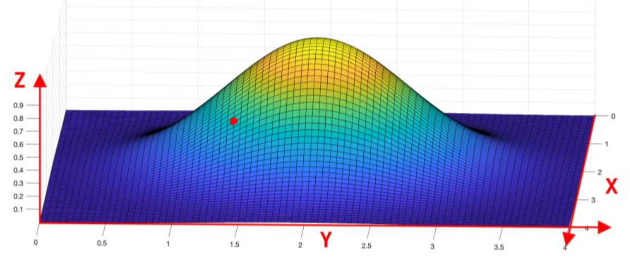


◯ -0.6

◯ 0

◯ 0.6

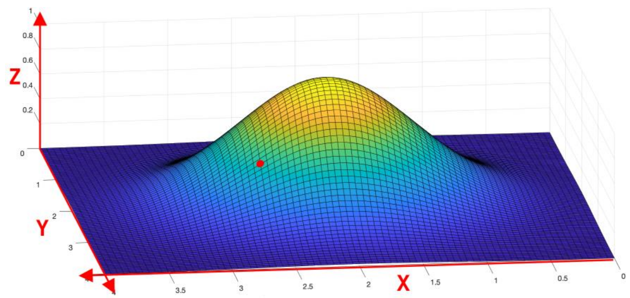
Derivative along y



◯ -0.6

◯ 0

◯ 0.6



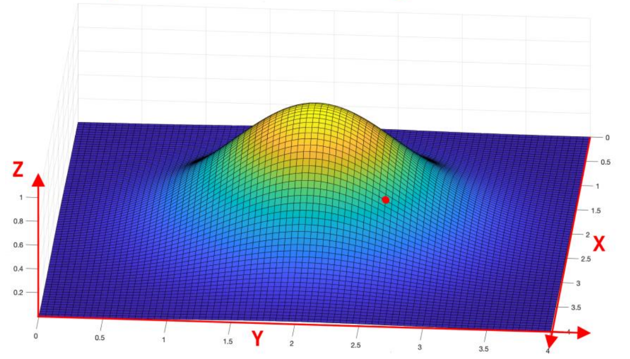
Derivative along x

◯ -0.6

◯ 0

◯ 0.6

Reverted axis

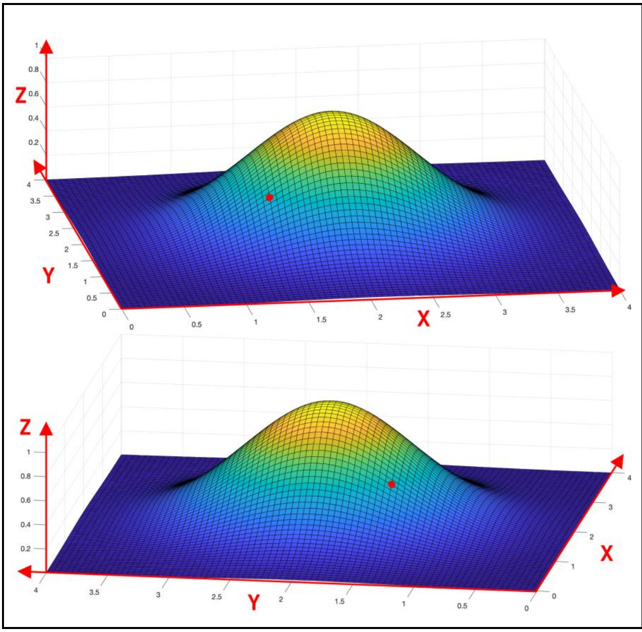


Derivative along y

◯ -0.6

◯ 0

◯ 0.6



Derivative along x

◯ -0.6

◯ 0

◯ 0.6

Derivative along y

◯ -0.6

◯ 0

◯ 0.6

Reverted axis

Questionnaire 2.6 (post)

Participant Id = ____________________

Please fill in the following questionnaire. This questionnaire is not part of your curriculum and will not influence your grade in any way.

Section 1.

How would you explain what a **derivative** is to your friend who doesn't know?

Section 2.

What do you think the value of the **derivative** is at the maximum of a **function**? Please justify your answer.

Section 3.

Do you think that two different **functions** can have the same **derivative**? Please justify your answer.

Do you think that it is possible to trace the graph of a **function** from the graph of its **derivative**? Please justify your answer.

Section 4.

Have you ever learnt about a concept similar to the **derivative** in another course? If yes, which concept and which course?

Questionnaire 2.7 (post)

Not included due to copyright.

The questionnaire contained elements 2, 3, 7, 8, and 9 from the Calculus Concept Inventory.

Reference

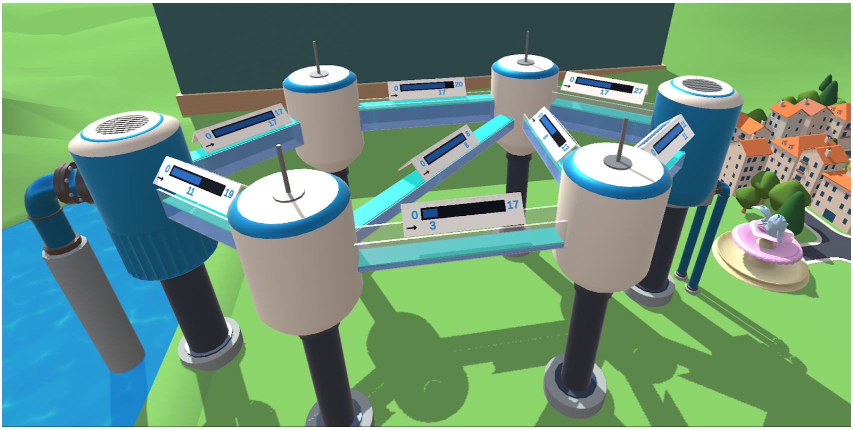
Epstein, J. (2007, September). Development and validation of the Calculus Concept Inventory. In Proceedings of the ninth international conference on mathematics education in a global community (Vol. 9, pp. 165-170). Charlotte, NC.

Obtained from (October 2021):
https://www.physport.org/assessments/assessment.cfm?A=CCI

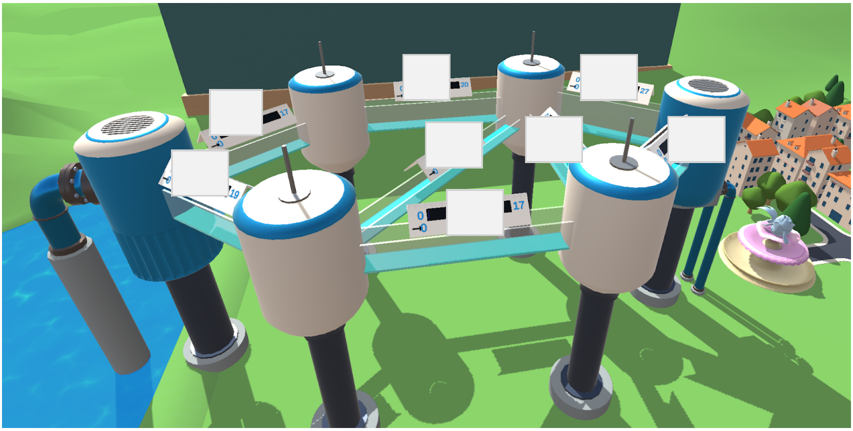
Id: _______________________________________

Transport as much water as possible from the lake to the town through the series of pipes below. Write the new amounts in the white boxes on the bottom picture.
If you think it is impossible to transport more water than what is in the first picture, cross this box: ☐

Initial situation:

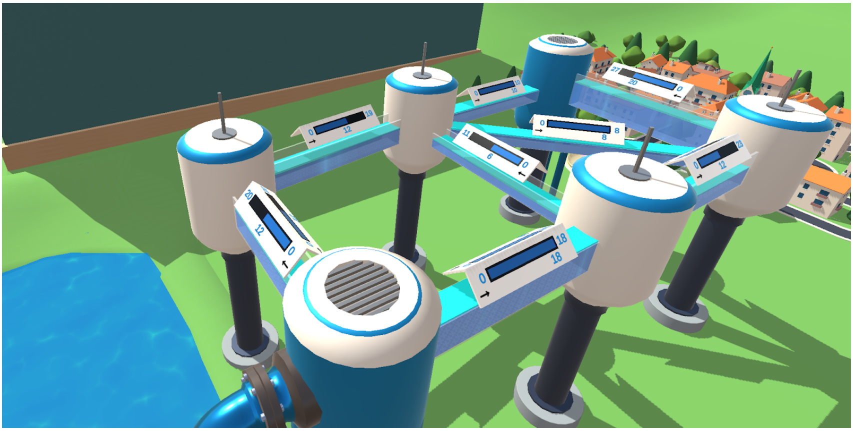


Your solution:

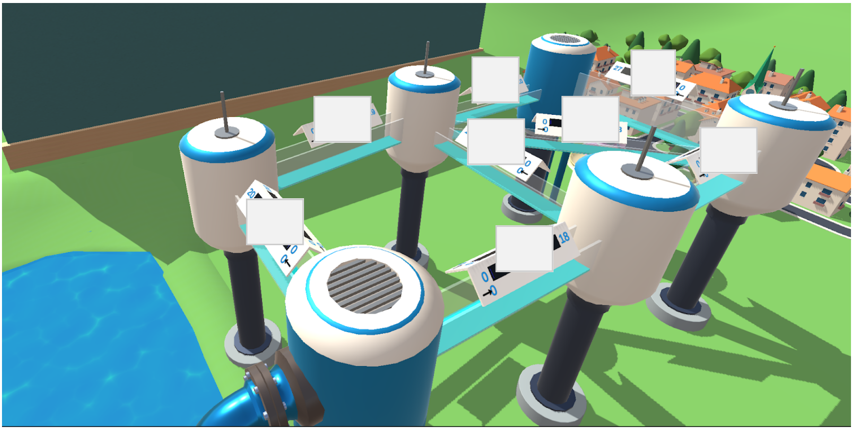


If you think it is impossible to transport more water than what is in the first picture, cross this box: ☐

Initial situation:



Your solution:



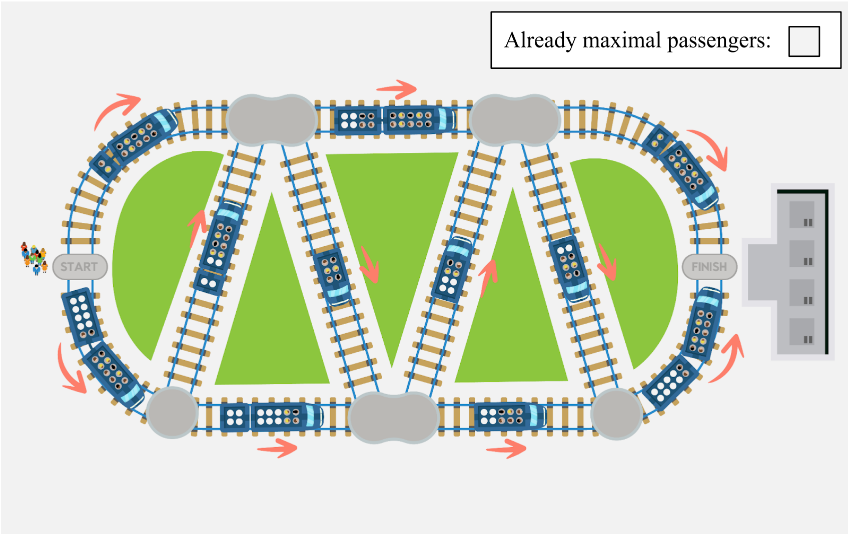
Id: ____

In the next two tasks you are the operator of a train network. People from all around the world would want to visit a beautiful town only reachable by your train network. Everyone will arrive at the station “Start” and will end their journey near the town at station “Finish”. When the train reaches the next station it will turn back and everyone has to board one of the next trains. Trains are timed so all of them arrive at the same time to their destination, there is time to board the next train and all of them will depart at the same time. Your goal is to allow as many people to visit the town as possible.
Tickets have already been sold through an online application denoted by the filled seats in the first picture, the white seats can still be sold to people. You can move people from their previously assigned trains to another if there are free seats available on the new train. People should not be left stranded on any of the middle stations.

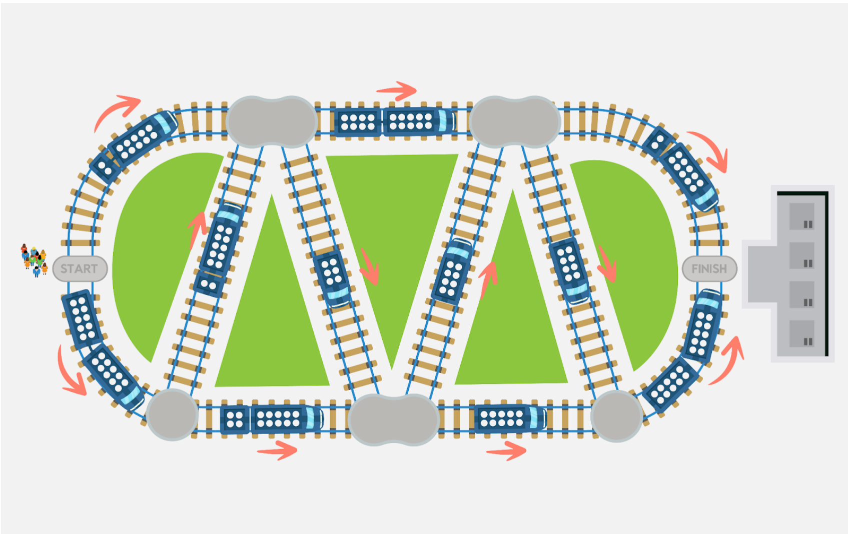
The first picture shows the current assignment. Please use the second picture to propose an assignment that maximizes the number of passengers getting to the city.

If you think it is impossible to allow more people to visit the town than those who registered online, cross the box on the top right corner of the respective image.

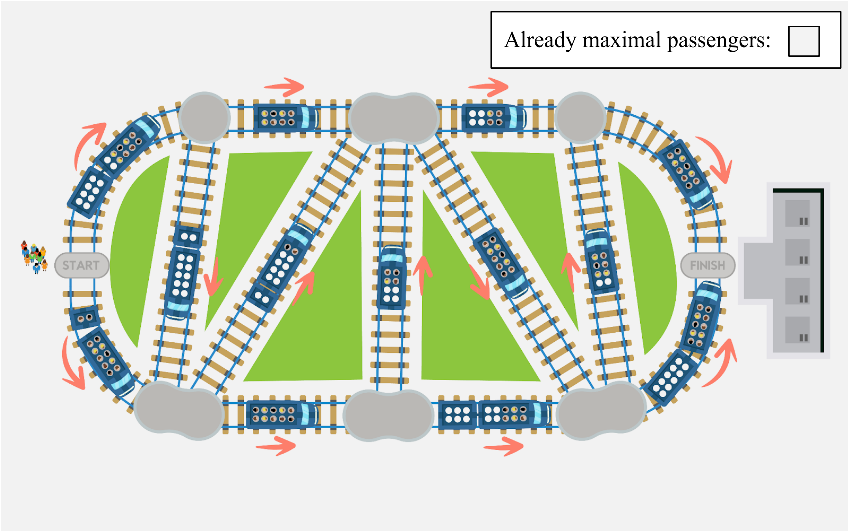
Initial situation:



Your solution:



Initial situation:

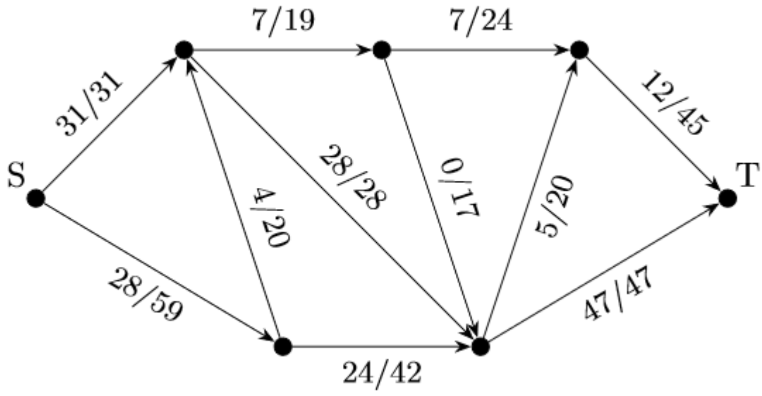


Your solution:

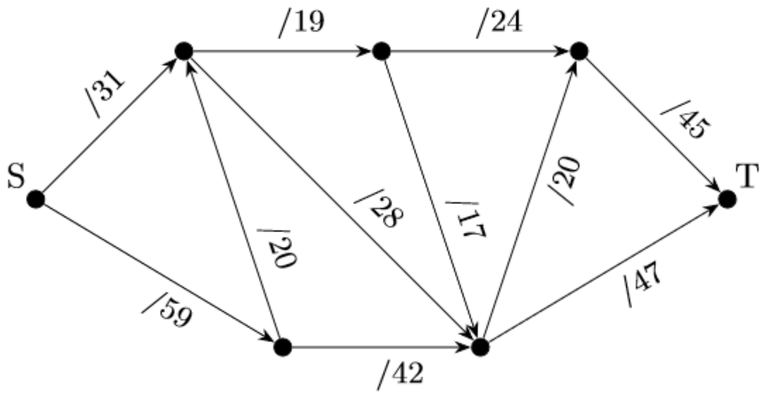
Id: ____

You are given a flow network on the top “Input Graph”. Increase the flow value of the network if possible. Write down the new flow values of the edges on the bottom network “Your solution”.

If you think it is not possible to increase the value cross this box: ☐

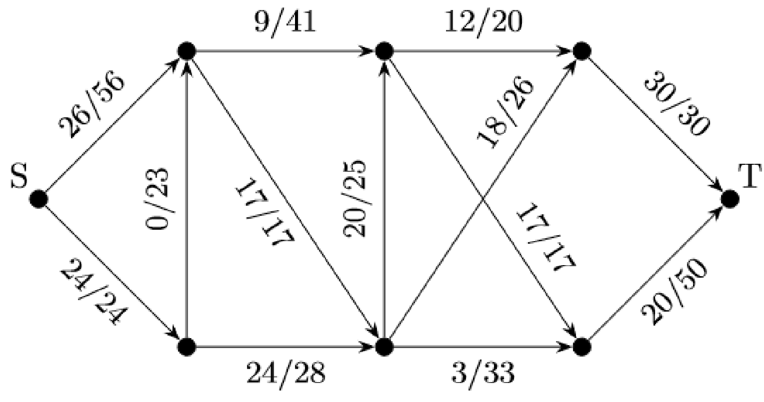


Input Graph

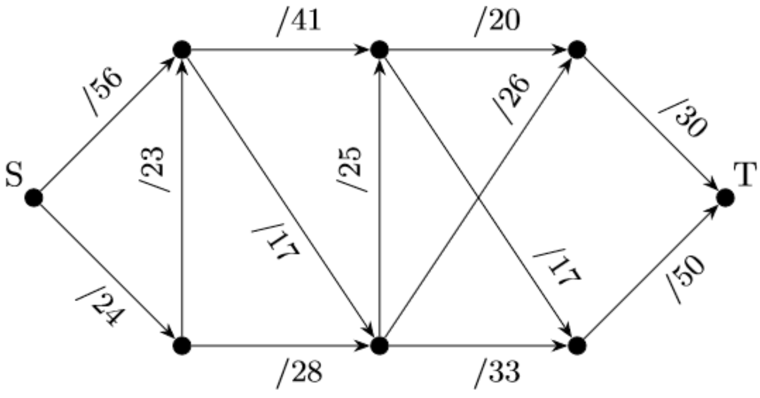


Your solution

If you think it is not possible to increase the value cross this box: ☐



Input Graph



Your solution